Exploiting Reduced Dimensionality in the Design and Control of Embodied Systems

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Abstract

High dimensionality poses several questions in our current understanding of the mechanisms of control and learning in nature. The key problem lies in understanding the principles by which nature copes with high-dimensional control and optimisation. Solving it is not only essential for a theory of motor control in embodied systems; it could also yield novel methods for the design and control of high-dimensional biomimetic artificial systems. In this thesis a possible solution to this problem is proposed in the form of a design principle of exploiting *reduced dimensionality*.

Reduced dimensionality is defined as the property of a system by which dimensionality reduction of its dynamics and behaviour is facilitated. This notion is formalised mathematically by a proposed control-theoretic framework of reduced dimensionality analysis. The framework requires the definition of a quality factor for the feasibility of the reduced dimensional model for control and the choice of an appropriate model reduction algorithm. Using this framework, the neuroscientific hypotheses of optimal motor control, developmental skill acquisition, and muscle synergies are analysed synthetically.

The framework is used for a systematic study of the different factors affecting reduced dimensionality , i.e. (i) natural dynamics, (ii) output (task-space relevance) and, (iii) input (modularisation of the control). Case studies further examine reduced dimensionality in the viewpoint of (iv) Dynamical Movement Primitives for robotics and, (v) dimensional change accompanying development of motor skills.

Both theoretical analyses and empirical demonstrations using simulations are presented for each of the studies. The results indicate that reduced dimensionality can be effectively exploited as a design principle for embodied systems. They have implications for both biological theories of motor control and development and for the design and control of high-dimensional artificial systems.

Zusammenfassung

Hohe Dimensionalität stellt uns vor mehrere Fragen in unserem gegenwärtigen Verständnis der Steuerungs- und Lernmechanismen in der Natur. Das Schlüsselproblem besteht im Verständnis der Prinzipien, dank denen die Natur hochdimensionale Steuerung und Optimierung bewältigen kann. Die Lösung dieses Problems ist nicht nur essenziell für eine Theorie der motorischen Steuerung in verkörperten Systemen; sie könnte zudem neue Methoden für Design und Steuerung von hochdimensionalen biomimetischen künstlichen Systemen ergeben. In dieser Doktorarbeit wird eine mögliche Lösung dieses Problems in Form von Designprinzipien, die *reduzierte Dimensionalität* ausnutzen, vorgeschlagen.

Reduzierte Dimensionalität ist definiert als die Eigenschaft eines Systems, durch die eine Reduzierung der Dimensionalität der Dynamik und des Verhaltens dieses Systems ermöglicht wird. Dieses Konzept wird durch das vorgeschlagene kontrolltheoretische Framework der *reduced dimensionality analysis* mathematisch formalisiert. Das Framework erfordert sowohl die Definition eines Qualitätsfaktors für die Umsetzbarkeit des Modells reduzierter Dimensionalität in der Regelungsstechnik, als auch die Wahl eines angemessenen Modellreduktions-Algorithmus. Über dieses Framework erfolgt dann die synthetische Analyse der neurowissenschaftlichen Hypothesen von optimaler motorischer Steuerung, von dem Erwerb von Fertigkeiten während der Entwicklung, sowie von Muskelsynergien.

Das Framework dient zur systematischen Untersuchung der verschiedenen Faktoren, die auf reduzierte Dimensionalität einwirken, d.h. (i) natürliche Dynamik, (ii) Ausgabe (Aufgabenraum-Relevanz) und (iii) Eingabe (Modularisierung der Steuerung). Fallstudien untersuchen reduzierte Dimensionalität ausserdem unter dem Gesichtspunkt von (iv) Dynamic Movement Primitives für Robotik und (v) dimensionale Änderung während der Entwicklung von motorischen Fähigkeiten.

Jede einzelne Untersuchung beinhaltet sowohl theoretische Analysen als auch empirische Demonstrationen. Die Ergebnisse machen kenntlich, dass reduzierte Dimensionalität effektiv als Designprinzip für verkörperte Systeme genutzt werden kann. Sie erlauben Folgerungen sowohl für biologische Theorien motorischer Steuerung und Entwicklung, als auch für das Design und die Steuerung hochdimensionaler künstlicher Systeme.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

"Less is More"

- Mies van der Rohe, Architect [1]

In our never-ending quest towards comprehending the nature of physical reality, one of the most enduring and challenging problems has been that of understanding intelligence and the principles underlying its emergence. This question has motivated a highly interdisciplinary study of intelligence that dates back to antiquity, while closely mirroring the history of philosophy. In recent times, research has been increasingly shaped by the dominant paradigm of embodiment - best captured by the phrase "*Intelligence requires a body*" [Pfeifer and Scheier, 2001]. Through the employment of the synthetic methodology, i.e. "*understanding by building*", the aim of embodied artificial intelligence has been the elucidation of the emergence of intelligence in an organism as a consequence of its physical embodiment within the environment. Towards this aim, a set of heuristic *design principles* have been proposed for artificial autonomous systems; such principles could potentially be used for the generation of empirical hypotheses for natural systems [Pfeifer and Scheier, 2001].

Adaptability and diversity of behaviour are two important characteristics of intelligent behaviour. From an evolutionary standpoint these two characteristics offer tremendous advantages in coping with the potentially adverse conditions affecting survival. The perspective of embodied AI is that these intertwined attributes can be manifested in an autonomous intelligent organism by endowing them with high-dimensionality in its neuro-mechanical substrate [Pfeifer and Scheier, 2001]. This is the redundancy principle in the sense of motor control - the presence of a large redundancy in the motor system allows an organism to successfully cope with uncertain and adverse situations. High-dimensionality however, raises the important question of what mechanisms underlie the emergence of motor coordination in the voluntary control of behaviour in systems despite the large neuro-mechanical redundancy.

The problems posed by large dimensionality are well-documented in engineering. In the traditional methods for robotics the complexity of computing real-time control solutions for high-dimensional systems is well known [Featherstone, 2007]. Although redundancy resolution methods have been proposed in robot control [Hollerbach and Suh, 1987], these methods often require extensive engineering effort for modelling and have so far only been applied to well-structured contexts such as industrial robotics. The insufficiency of traditional control methods is increasingly coming into focus in novel approaches to robot designs such as biomimetics, where the problems of large dimensionality in the morphology is accompanied by modelling difficulties.

[1] Most famously refers to van der Rohe's philosophy of minimalism.

Although machine learning and optimisation methods have been used for autonomously endowing robots with control abilities, the difficulties of large dimensionality still persist. The notorious aphorism "*Curse of dimensionality*" was coined by Bellman [Bellman, 1961] to describe the various problem that render learning in high dimensions intractable. Viewed from a control perspective, although redundancy increases the space of potential solutions, the state space itself might be too large to explore within a reasonable time frame. An alternative is to look to nature in trying to understand how motor coordination may be self-acquired autonomously and gradually; this is the approach of developmental robotics [Asada et al., 2001].

From a biological perspective the high-dimensionality problem can be viewed in terms of the three time scales : (i) phylogenetic, (ii) ontogenetic and (iii) here-and-now perspectives [Pfeifer and Bongard, 2007]. From a phylogenetic viewpoint, redundancy can be considered beneficial since it affords an organism a multitude of possibilities for accomplishing tasks. However it has been hypothesised that the Darwinian principles might also apply to the control of behaviour since this directly impacts the fitness of an organism; sub-classes of behaviour such as movement coordination thus are surmised to be dictated by optimisation principles [Harris and Wolpert, 2006, Todorov, 2004]. For an individual organism, this view translates into a question of how optimal solutions might be self-acquired within the high-dimensional space of possibilities during its ontogenetic development. There is a definite advantage in acquiring the solution within a short time frame since this increases the chance of survival of the organism and therefore the fitness. Furthermore, this is not a static optimisation problem; it has been surmised that some developmental adaptation mechanism must underlie this seemingly life-long quest for optimal behaviour [Harris, 2011]. Thus, large dimensionality appears to be a double-edged sword, providing behavioural adaptability at the price of ontogenetic learnability.

In the shortest time scale of here-and-now, high-dimensionality also raises another important question; how many of the repetitive behaviours in nature arise from only a small subset of possibilities. Regardless of morphological differences, individuals within any species often demonstrate stereotyped motor behaviours. It has been incorrectly surmised in the past that this is indicative of some form of stored pattern mechanism achieving coordination; however there is plenty of evidence for context-dependent behavioural variability [Turvey et al., 1982] and kinematic regularities with invariant characteristics. For example, human motor behaviour has been characterised by movements that seemingly follow empirical observations such as the Fitt's Law (speed vs. accuracy tradeoff), and optimal smoothness (minimising the rate of change of accelerations, torque changes or variability). The presence of such invariants in motor behaviour seems to indicate that natural systems are solving the control problem repeatedly by utilising some underlying principles rather than by mere repeating predefined solutions [Todorov, 2004].

Thus, the motor coordination problem lies in understanding how nature circumvents the apparent difficulties of learning and control in high-dimensions so that the redundancy may be optimally exploited. From a situated perspective of an autonomous agent the questions are: what are the various ways to act in a given situation? How to decide what is the appropriate action within a reasonable amount of time, in the face of constraints? Fortunately, we are not completely in the dark in our understanding this problem. There is a growing consensus among motor-neuroscientists that the mechanism employed in complexity resolution is dimensionality reduction in behaviours (stereotypy) and in their neural control (modularity). This thesis focuses on understanding this principle of reduced dimensionality through a synthetic approach within a consistent mathematical framework.

1.1 Motivation and Research Problem

The research presented in this thesis is motivated by two main factors : (i) the need for novel approaches in Robotics and AI for tackling problems of large dimensionality and, (ii) the need for a synthetic approach in validating unresolved neuroscientific hypotheses on motor coordination.

In robotics and AI, a paradigm shift has taken place in the form of bio-inspiration and biomimesis. This has lead to an explosion of novel technology especially in the morphologies and materials involved in the design of robots; some of these developments are surveyed in Sec. 2.3. Two current research areas which can be considered to be very promising are biomimetics and developmental robotics. Biomimetics has lead to development of robot morphologies with muscle-like actuators and multimodal sensing. Progress in this field has however has been hindered by the inability of traditional approaches in solving the high-dimensional control problem. Developmental robotics, on the other hand, has looked at ontogenetic development in nature as a source of inspiration for self-learning artificial systems. Although exciting in its prospects, the approaches of developmental robotics have so far not successfully scaled up to tackle high-dimensional morphologies such as anthropomimetic robots. Clearly there exists a need for a sound theoretical approach in coping with the problems posed by large-dimensionality by taking inspiration from the developmental self-organisation of motor control. This approach could therefore lead to novel design principles for biomimetic robot control.

From the biological viewpoint, the well-known *Degree of Freedom* (DoF) problem was identified by Russian neuroscientist Nikolai Bernstein as being critical to understanding the underlying principles of biological motor control [Bernstein, 1967]. There is substantial evidence for neural control strategies that reduce dimensionality. As reviewed in detail in Sec. 2.2, a number of different hypotheses and models have attempted to explain the underlying mechanisms. In the here-and-now perspective, computational models of the motor control mechanism have suggested that redundancy resolution and motor coordination arise out of a system of paired forward-inverse models, or through modulation of a network of reflexive feedback pathways. Statistical regularities in the motor behaviour on the other hand suggest that the mechanism involves a modularisation of the control problem; the hypotheses of motor primitives and muscle synergies in the CNS reflect this view. From a developmental perspective, theories have been proposed that suggest there is a progressive exploration of sensorimotor space, arising from a mechanism of DoF freezing and unfreezing. The evolutionary view of this problem is echoed in proposals for optimisation principles in motor control, as suggested by the presence of a number of kinematic invariants in motor behaviour. Given the scope of each of these theories, there is a need to address the underlying problem using a consistent theoretical framework in order to validate the existing hypotheses. Consequently a synthetic approach in quantifying and solving the DoF problem might be beneficial in tackling the unresolved questions.

Motivated by these needs, in this thesis it is proposed that the problems of high-dimensionality can be solved through the exploitation of reduced dimensionality in the design and control of a system. Reduced dimensionality (defined in the next section) denotes the properties of a system that facilitate the dimensionality reduction of its behaviour. The primary aim of this thesis is to investigate the following research question :

What is reduced dimensionality and how can it be exploited for the design and control of embodied systems?

Towards realising this aim, the primary requirement is the formalisation of the question : *what is reduced dimensionality?* Once this notion can be quantified, the solution to its exploitation entails a systematic analysis of the various causative factors within a system that enable reduced dimensionality. The results then point towards quantified design principles for the exploitation of reduced dimensionality, consequently leading to novel methods for robotics as well as testable hypotheses for neuroscience. The proposed framework of reduced dimensionality analysis is introduced next.

1.2 Reduced Dimensionality Analysis

In seeking design principles that underly natural biological phenomena, a consistent mathematical framework is essential for quantification. Before the proposed framework is introduced, it important to distinguish the terms *Dimensionality Reduction* (DR) from *Reduced Dimensionality* (RD).

Note on Terminologies

For the research presented in this thesis, the term '*behaviour*' is used within its meaning in physics and engineering, i.e. simply trajectory of a system in response to actuations. Therefore, a representation of behaviour henceforth denotes an input-output model of a given system as seen from an external perspective. The generality of the results from the proposed analysis with respect to embodiment is discussed again in Chap. 9.

1.2.1 Dimensionality Reduction vs. Reduced Dimensionality

DR denotes the process or methods of reducing the dimensionality of a dataset or of a system by exploiting some mathematical properties within, such as statistical regularities or structure. In the statistical and machine learning context, dimensionality denotes some measure of the number of parameters or units (features) that characterise a set of observations (data). Redundancy in this context means that these features are not independent of each other. DR therefore aims at summarising the large set of features and parameters into a smaller set with little or no redundancy [Lee and Verleysen, 2007]. Common applications for such methods include data analysis, optimisation, and robotics etc.

In a control theoretic perspective, DR refers to the process of finding a representation of the *dynamics* of a given system that reduces the number of dimensions while preserving certain properties. Descriptions of the dynamics of systems are often provided by using natural laws. These tend to produce verbose descriptions in terms of number of variables that are used to describe observed behaviour. The control-theoretic perspective is used to distinguish a set of variables as the (i) *input* (which can be directly influenced) and the (ii) *output* (which can quantify the intention of the control) from the variables describing the behaviour of the system itself which is usually the (iii) *state*. Dimensionality in this context refers to the size of the state describing the behaviour.

DR in a control perspective is aimed at reducing the number of state variables needed to specify behaviour, while preserving the input-output relationship to within acceptable limits. The algorithms and methods for this control-DR are studied in the control engineering discipline of Model Order Reduction (MOR) [Antoulas et al., 2001]. The broad framework is depicted in Fig. 1.1 while Sec. 2.4 reviews many of these methods in detail.

On the other hand, RD is not a method or algorithm, but rather denotes a property of a given system. The following broad definition is proposed within the scope of this thesis :

Reduced Dimensionality : *The property of a high-dimensional system facilitating the dimensionality reduction of its dynamics and behaviour.*

Reduced dimensionality enables dynamic models of the system to exist which are lower in dimensions to the system itself. Although such models cannot replicate the input-output relation to the same degree as the original system, provided the loss of information is within acceptable limits for a given task, they might be adequate. Hence, the reduced dimensionality of a system is inherently task-dependent.

Furthermore, from the viewpoint of the theory of embodiment, the diversity of behaviour of the system results from its inherent (natural) dimensionality. Conversely, reduced dimensionality implies that task-specific simpler models of the behaviour may instead be utilised for learning control. Thus, reduced dimensionality facilitates task-specific control simplification.

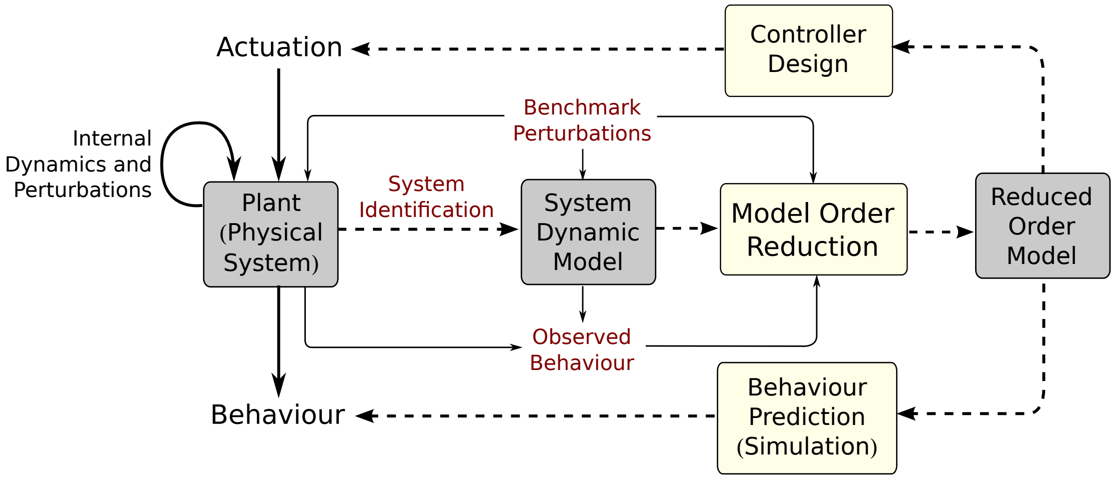

Figure 1.1: Model Order Reduction (MOR) : control-theoretic perspective on dimensionality reduction of a system

1.2.2 Framework Description

The framework that is proposed in this thesis quantifies reduced dimensionality if a system using the control-theoretic perspective of Model Order Reduction (MOR).

The main elements of this framework are depicted in Fig. 1.2. First a state-space dynamic model of the system's behaviour is developed by using modelling and system identification techniques. The model which describes the relationship between the actuation signals (input), internal and external dynamics (state), and the outcome which is the behaviour (output). The choice of reduction algorithm must be specified; MOR algorithms based on the projection framework are used (see Sec. 2.4). A task-specific quality factor is used to define the acceptability of the reduced dimensional model, the quantification uses evaluations of control performance on benchmark tasks. MOR algorithms are then used to derive equivalent reduced dimensional dynamical representation of the behaviour. This equivalent representation best preserves the behaviour of the original system, as defined by a quality measure. The reduced dimensionality is a measure of the dimensionality of this equivalent reduced dimensional system.

The mathematical formulation of the framework of Fig. 1.1 is described in Chap. 3. It is used to analyse the *reduced dimensional* behaviour of a system through utilisation of model and control dimensionality reduction methods. A more formal discussion of the framework is presented in Chap. 8.

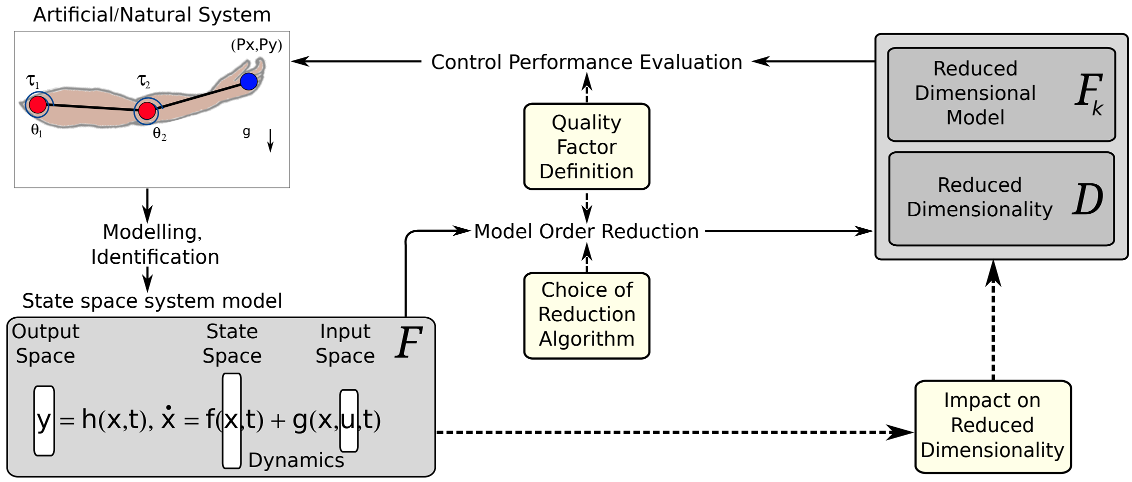

Figure 1.2: Reduced dimensionality analysis : control-theoretic mathematical framework for the study of factors impacting reduced dimensionality of a system. The framework requires the definition of a quality factor for quantifying the feasibility of the reduced dimensional model for control and the choice of an appropriate model reduction algorithm.

In the case studies presented in this thesis, two techniques for dimensionality reduction are mainly employed : (i) Proper Orthogonal Decomposition (POD) and (ii) Balanced Reduction (BR). The former utilises statistical properties in a system to reduce the dimensionality while the latter exploits the relationship of the state space to the input and output space for a given task. In either case, an ideal measure for quantifying the *reduced dimensionality* can be derived, namely that of (i) Proper Orthogonal Mode magnitudes (POM), and (ii) Hankel Singular Values (HSV) respectively.

It must be noted that this thesis does not aim to develop new algorithms for DR itself, such methods are the domain of MOR [Antoulas et al., 2001]. The novel contribution of this framework lies in harnessing the algorithms in order to develop methods by which reduced dimensionality may be quantified and therefore exploited in embodied systems – both in biological models and in artificial examples. A summary of the main studies carried out in this thesis follows.

1.3 Summary of Studies

This thesis aims at systematically studying the phenomenon of reduced dimensionality and deriving methods for its exploitation in a synthetic approach. The development of the mathematical framework is the primary contribution. Five studies are carried out exploring the exploitation of reduced dimensionality, and are presented in individual chapters as described in Fig. 1.3.

1.3.1 Natural Dynamics and Reduced Dimensionality

In the first study the relationship between the natural dynamics and reduced dimensionality is explored using two ways.

The motor primitive hypothesis is cited as an example of input dimensionality reduction. It has been proposed that the number of primitives is related to the reduced dimensionality in a system. Furthermore, developmental underpinnings have been tested in the form of increasing number of primitives accompanying growth and development of an organism. Quantifying the

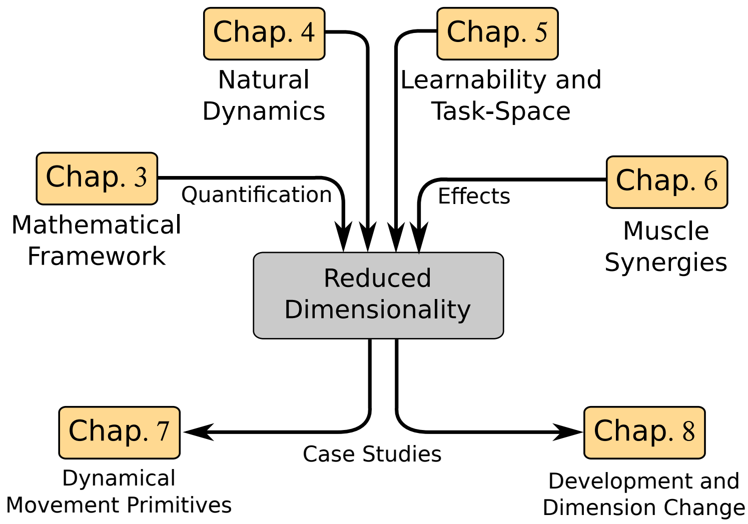

Figure 1.3: Exploiting reduced dimensionality : studies carried out and their chapter organisation in this thesis.

relationship of the natural dynamics to the reduced dimensionality is essential in understanding how development is related to the actual process of growth in the physical substrate. The first demonstration examines the variation in the reduced dimensionality due to variation in the passive mechanical properties. Using a simulated high-dimensional chain of mass-spring-damper elements as a loose analogy of a vertebrate limb, demonstrations are presented to show how certain kinds of configurations of passive properties facilitate reduced dimensionality to a greater degree.

The POD method is then used to show that the model dimensionality that captures the behaviour of this system can be regulated through appropriate choice of passive properties – therefore enabling ease of control. These results are relevant to the understanding of the role played by the passive mechanical properties of the body during the developmental process of motor skill acquisition. Since growth and maturation affects these passive mechanical properties, the results of this study can be viewed as a preliminary exploration of the ideal '*growth*' directions in a parametric space that an organism could follow in order to exploit reduced dimensionality.

The second part of the study presents a synthesis model for a reduced dimensional controller inspired by the biological notion of motor primitives for a redundant compliant robot. Influenced by the biological phenomenon of spontaneous motor activity, a developmental methodology is presented for the unsupervised acquisition of motor primitives. The proposed method utilises POD for reduction and is numerically simple. The synthesised primitives represent a form of dimensionality reduction of the control inputs. The simulation experiments on the pendulum robot model show how the proposed controller generalises to unexplored regions of the workspace using the knowledge of the equilibrium positions resulting from the individual primitives. From a reduced dimensionality perspective, a key observation that can be made is that the number of primitives that is required can be directly obtained by the reduction in dimensionality of the system dynamics; this in turn results from the passive mechanical properties of the system, i.e. the natural dynamics. The outcome of this demonstration is a developmental control synthesis method that can be used for biomimetic robots.

1.3.2 Learnability, Task Space, and Reduced Dimensionality

This study presents two key results relevant to developmental psychology and robotics. In the first, a theoretical equivalence is established between the hypothesis of motor primitives and norm-minimising optimal control. It is shown theoretically how reduced dimensionality underlies these two principal classes of approaches to motor control. The notion of *learnability* is then introduced as a limit on the rate of learning during development due to the constraints of high-dimensionality. Learnability is therefore a direct consequence of reduced dimensionality in a system.

In the second demonstration, Bernstein's developmental model for progressive DoF increase through unfreezing is re-examined from the perspective of the reduced dimensionality analysis framework. A nonlinear dimensionality reduction method of empirical balancing is used to extract reduced dimensional representations. This is demonstrated on a nonlinear model of a vertebrate limb that incorporates muscle dynamics. The concept of progressive dimensional increase is then quantified in the perspective of reduced dimensionality by showing how task-specific improvements of performance can result during development.

1.3.3 Muscle Synergies and Reduced Dimensionality

The motor neuroscience hypothesis of muscle synergies is often cited as the most direct evidence for reduced dimensionality principles underlying motor control. A muscle synergy is defined as a coordinated activation of groups of muscles. However, there are open questions remaining in proving the validity of this hypothesis. Moreover,the links between the synergy hypothesis, optimal motor control and development are yet unclear. This study addresses the prevalent criticisms by developing a bottom-up method to gauge the validity of this hypothesis.

The question 'can muscle synergies reduce the dimensionality of behaviour', is examined in detail using the temporal muscle synergies formulation - control is composed of a task-specific weighted linear combination of task-independent patterns. Using the analysis framework of this thesis, two important methods are derived for investigation of this problem : (i) Trajectory Specific Dimensionality Analysis (TSDA) and (ii) Minimum Dimensional Control (MDC). For a given task and a given set of synergies on a system, TSDA quantifies the reduction in dimensionality of the system following a trajectory that satisfies the task constraints. A reduced dimensionality measure using Hankel Singular Values (HSVs) is derived for this purpose. MDC utilises the TSDA for computing the optimal trajectory and the corresponding weights on a given synergy set, that minimise dimensionality while achieving task objectives. A suitable cost function is derived from the HSV based measure of the reduced dimensionality in the system.

Simulation results on both linear and nonlinear systems, utilising these methods, show that straight smooth sigmoidal trajectories emerge as the optimal dimensional trajectory for the reaching task. The results are very close to experimental observations of human behaviour and the proposed methods can therefore potentially be used to synthetically validate experimentally extracted synergies.

1.3.4 Dynamical Movement Primitives and Reduced Dimensionality

The fourth study extends the framework of TSDA for analysing the reduced dimensionality in behaviour due to employment of Dynamical Movement Primitives (DMP) for control. The DMP control strategy consists of a set of tunable nonlinear dynamical systems that can be used to coordinate movement. The proposed method allows the quantification of the reduced dimensionality by utilising an analytical formulation of the solution to the DMP equations. Iterative Basis Extraction (IBE), a numerical algorithm to obtain the basis set of the DMP is developed for converting a system under DMP control into the temporal synergy formulation; TSDA can then be employed to quantify the reduced dimensionality. Using this approach, various trajectories followed by a system are compared in terms of reduced dimensionality and the reduced dimensional models that are extracted are analysed qualitatively for their accuracy. Experiments are performed on simulated linear (n-element mass-spring-damper chain) and nonlinear (compliant kinematic chain) systems to demonstrate how some trajectories are seemingly lower in dimensionality and thus appropriate task-specific dynamic models of the behaviour can be synthesised.

1.3.5 Development and Dimensional Change

The fifth and final study of this thesis presents a mathematical formalisation of the phenomenon of dimensional change during development by using the reduced dimensionality framework. It is shown both theoretically and empirically how dimensional change can be achieved through parametric variations in a system and results in progressive acquisition of increasingly complex skills. The concept is then tested on a model of vertebrate limbs to demonstrate that optimal "paths" in parametric space can exist, wherein growth can regulate the learnability, i.e. through regulation of the reduced dimensionality.

1.4 Thesis Scope

This thesis presents a set of system case studies of the various factors which impact the reduced dimensionality. It should be kept in mind that many of these results are of a proof-of-concept nature rather than realistic biological models or mature robot algorithms. This is a deliberate choice since the model systems presented here can be studied more thoroughly with analytic tools. Similarly, the focus is only on some kinds of '*simple*' behaviours since they allow easy characterisation in terms of kinematic and dynamic descriptions.

1.4.1 Model Systems

The choice of the level of abstraction in the physical models for motor control studies is critical as it can complicate the analysis of the results obtained from testing hypotheses [Valero-Cuevas et al., 2009]. In this thesis a set of abstract idealised physical systems are employed to test the theoretical predictions. Broadly, the models studied are :

1. Linear systems in $1D$: mass-spring-damper systems with varying number of elements
2. Linear systems in $2D$: pendulum robot simulation with actuator dynamics, tethered mass system and the eye movement model
3. Planar kinematic chain : rigid limb in gravity, compliant limb, compliant limb with muscle models

The mass-spring-damper chain is a well-understood model often employed in physics and control theory to study coupled oscillatory phenomena and some kinds of mechanical systems. The masses in this system are constrained to move along $1D$. This system was used for studies of model reduction in order to understand the role played by passive properties in the reduced dimensionality.

The second system uses a mass that is anchored to a point on a plane with passive forces. This kind of a system is is loosely inspired by the mechanics of the oculomotor system, which consists of an orb anchored by weak passive forces and is actuated by opposing muscles in 2 orthogonal directions. A variant of this system was used to model the dynamics of the pendulum robot by incorporating linear actuator dynamics to simulate the series elastic actuation. This system was used in experiments for synergy synthesis.

The third system is a nonlinear model that is often used in robotics and in neuroscience for understanding limb dynamics. Variations of this system utilised gravity, passive compliance at the joints, series elastic actuation at the joints, and passive joint compliance. This system was used to demonstrate the reduced dimensionality in nonlinear systems.

In terms of actuation, in some of the biologically oriented experiments a linear dynamic muscle model is used. This simplifies the analysis, while simultaneously increasing model dimensionality. In the robotic simulation experiments, a simple actuator model simulating series-elastic actuators was used. Also, in terms of nonlinearities, the focus is on the trigonometric nonlinearity of the kinematic chain. Nonlinear effects such as contact dynamics, joint limits, and actuator limits, are ignored in these experiments. It must be noted that this is not a limitation of the proposed methods since the algorithms can cope with the introduced nonlinearities. Rather it is a deliberate choice that facilitates a more thorough analysis.

1.4.2 Behaviours under study

In thesis mainly two kinds of tasks or behaviours are examined : (i) point-to-point *reaching* and (ii) periodic motion. This is motivated by the argument that evolutionary fitness of an individual organism in terms of its survivability is not only dependent on occasional critical behaviour, but also on the control performance in the simple behaviours that are frequently necessary [Harris and Wolpert, 2006]. Simpler behaviours are also easier to characterise in terms of kinematics and dynamics; thus control performance can be measured objectively.

Reaching is one of the most important of early motor behaviours observed in human infants [Shadmehr and Wise, 2005]. Reaching is critical for sensorimotor development since it enables an infant to explore and interact directly with the world before locomotion can take place. The behaviour requires the usage of the entire limb and thus accurate reaching performance necessitates coordination. A related form of reaching of equal importance concerns saccadic eye movement, which bring a target to focus in the retina. Although there is no direct interaction with the environment, this is a critical behaviour for survival since eye movements are a fundamental feature of our perceptual abilities. Saccadic motions are also an interesting phenomenon for study since they are frequent, and their speed implies that control mechanisms must act in a feedforward manner.

From a robotic viewpoint, although reaching may be considered a relatively simple behaviour it is a useful case study for testing manipulation. More complex tasks are outside the scope of the presented results.

1.5 Thesis Organisation

This thesis is organised as follows : Chap. 2 presents a broad literature survey relevant to reduced dimensionality from the perspectives of biological motor control, robotics and AI, and control theory. A case is built up for the need for a consolidated theoretical framework in which dimensionality can be studied.

The studies carried out in this thesis are summarised within the mathematical framework of reduced dimensionality analysis in Chap. 3. The first study on the effect of natural dynamics is examined in Chap. 4. In Chap. 5, the second study relating learnability, task space and reduced dimensionality is described.

Chap. 6 describes the third study of this thesis : relating the muscle synergy hypothesis to reduced dimensionality in behaviour using the Trajectory Specific Dimensionality Analysis and Minimum Dimensional Control proposals. In the fourth study, presented in Chap. 7, the reduced dimensionality due to a control strategy using Dynamical Movement Primitves (DMP) is quantified by using the TSDA approach. Simulation results demonstrate how task-specific dimensionality reduction can be achieved when using DMPs for control.

The fifth and final study, presented in Chap. 8, consolidates the earlier results into a unified formal mathematical framework for the analysis of dimensional change. This is followed by a discussion of the implications from multiple perspectives, the future outlook and limitations of this work and the conclusions in Chap. 9.

Chapter 2

Research Background

In this chapter, the notion of reduction of dimensionality for motor control is reviewed from three perspectives of three areas : neuroscience, robotics and control theory. First, dimensionality in the context of motor control is defined mathematically. Dimensionality reduction is introduced as an approach towards simplifying analysis and control.

In neuroscience, the dimensionality problem and its mitigation has been addressed in hypotheses for motor control and in the developmental acquisition of motor skills. In robotics, the problems posed by dimensionality problem are well known but are dealt with only implicitly so far. Recent research areas such as under-actuated and biomimetic robotics, developmental robotics have tackled the dimensionality problem control and learning. Lastly, in control theory, algorithms for synthesising reduced dimensional models and controllers for systems have been developed. The applications are in the simulation, prediction and control of behaviour of complex systems in engineering.

An important objective of this chapter is to establish the need for a consistent theoretic framework for the analysis of reduced dimensionality and its reduction from the viewpoint of expediting the learning and optimisation of motor control in embodied systems.

2.1 What is Dimensionality?

The term dimensionality is used widely in a variety of contexts. Loosely speaking, it refers to the number of independent dimensions (degrees of freedom, coordinates, parameters) that need to be specified to fully determine a quantity or a system of quantities, such as the physical state of a particle or object, a statistic (other examples). Dimensionality is often considered as a measure of the overall a system's complexity. When describing the complexity of behaviour of a system, dimensionality is used in both, a spatial (number of components) as well as temporal (bandwidth) sense.

Coping with the problems posed by high-dimensionality in terms of the neuroscientific study of computational motor control [Wolpert, 1997,Flash and Sejnowski, 2001]. The problem of motor control is inherently ill-posed. There are nearly infinite number of solutions by which the motor system can potentially perform a task - thus discovering the redundancy resolution is one primary objective of motor control research.

In machine learning, and statistics, the problems of large dimensionality are well known; the phrase "*Curse of dimensionality*", coined by Bellman captures the difficulties faced in handling high-dimensional learning problems [Bellman, 1961]. Some of the well known effects of high-dimensional learning problems include sparsity of learning data, inadequate distance metrics, and the need for many expensive evaluations of objective function. In neuroscientific study of motor control, dimensionality can be dealt with in a wide variety of levels, right from the lowest in terms of physical movements to the highest cognitive concepts such as language learning and emotions. From a control perspective, dimensionality usually measures the complexity of the control problem, although not all physical dimensions need to be directly accessible or observable – this is dependent on the tasks that need to be performed instead. Here we consider dimensionality from the neuro-mechanical viewpoint of a single organism executing behavioural tasks, such as walking, reaching, etc. First the mathematical definition for the dimensionality of a dynamical system is summarised briefly

2.1.1 Mathematical Description

In mathematics, the dimensionality of a vector space is simply the number of elements present in any basis of that space. This me the number of coordinates necessary to specify any vector belonging to that space - i.e. the cardinality of a basis [Strang, 2003]. Geometrically, the dimensionality of an object is a topological measure of the size of the properties defining the object. Dimensionality measures the number of coordinates needed to specify a point on the object. For example, a point is zero-dimensional, a disc is two-dimensional, while a ball is three-dimensional. This geometric definition can be extended to quantify behaviour in the spatial sense of dynamical system, or in a temporal sense of signals.

Dimensionality of Dynamical Systems

For a dynamical system, dimensionality is a measure of the size of its state-space. If the system can be described by Ordinary Differential Equations (ODE), then its dimensionality refers to the order of the ODE [Braun, 1993]. For e.g., consider the ODE,

$$x^{(n)} = \mathrm{F}\left(t, x, \dot{x}, \ddot{x}, \cdots x^{(n-1)}\right), \tag{2.1}$$

where x is a function of t, $\dot{x} = \mathrm{dx/dt}$ is its first derivative with respect to t, and so on until, $x^(n)) = \mathrm{d^n x/dt^n}$, the n^{th} derivative with respect to t. The order of the system is then simply n, i.e. the order of the highest derivative. The number of inital conditions needed to be provided for a numerical solution to this ODE is also the same as the order.

Also, any ODE of order n, such as Eq. (2.1), can be written as a system of n first-order ODEs, such that the new system of equations is,

$$\dot{\mathbf{x}} = \mathbf{F}(t, \mathbf{x}), \tag{2.2}$$

where $\mathbf{x} = [x_1, x_2, \ldots x_n]^T$ and $\mathbf{F}$ is now a vector valued function. In this system, the dimensionality is given by the number of dimensions of the vector $\mathbf{x} \in \mathbb{R}^n$, i.e. same as the order of Eq. (2.1), n. An important aspect of this definition is that dimensionality, as defined on a set of 1^{st} order ODEs, is always a positive integer.

This mathematical formalism is utilised in the context of control theory and state-space methods [Dorf, 1991], where the dynamics are typically expressed as a set of ODEs and the dimensionality depends on the number of such equations specifying the input-output relationship. The general form of the state-space model is given by,

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}, \mathbf{u}, t), \quad \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, t), \tag{2.3}$$

where $\mathbf{x} \in \mathbb{R}^N$ is denoted as the state , $\mathbf{u} \in \mathbb{R}^I$ is denoted the input and $\mathbf{y} \in \mathbb{R}^O$ the output. By this definition, N is the *state dimensionality*, *input dimensionality* refers to I and O is the *output dimensionality*. Again, by definition, $N, I, O \in \mathbb{Z}^+$, i.e. the space of positive integers.

The notion of Degree of Freedom (DoF) is closely related to the dimensionality of a system. In robotics and in movement neuroscience, it refers to the dimensionality of the configuration space of a manipulator or a limb [Spong et al., 2006]. The phase-space dimensionality of the dynamics of a robot manipulator, is typically twice the DoF of the manipulator (under rigid-body dynamics assumptions) since it incorporates both position and momentum variables.

Temporal Dimensionality of Signals

Dimensionality is also utilised in a temporal sense in the description of the behaviour using signals. Each dimension of a physical system of the form in Eq. (2.3) is also a function of time that can, in principle, be manipulated independently of the other dimensions. From sampling theory, the dimensions of a time varying bandwidth-limited real function is approximately given by the Shannon number S as,

$$S = 2WT, \tag{2.4}$$

where W is the one-sided bandwidth and T is the duration of the signal. This number is often considered to be a fixed quantity for a given signal. However, when used to describe the temporal dimensionality of behaviours such as in human movements, this is imprecise as many behaviours have finite duration (and some are very brief). Strictly speaking, the signals corresponding to these behaviours must therefore have infinite bandwidth, although Fourier energy does fall off rapidly at high frequencies. In the 1960s, dimensionality has been made more precise for almost-bandwidth-limited signals [Slepian and Pollak, 1961,Landau and Pollak, 1961,Landau and Pollak, 1962]. The key point is that dimensionality of the evoked is traded-off against accuracy. More accuracy in control requires more bandwidth, and hence higher dimensionality in the controller.

2.1.2 Dimensionality Reduction

When trying to study systems through the signals produced, high-dimensionality complicates the analysis. The aim of dimensionality reduction is to reduce the number of variables under consideration, while keeping the information loss to a minimum [Lee and Verleysen, 2007]. This simplifies the analysis of complex systems and enables the handling of high-dimensional training data. A common application of such techniques is for simplifying analysis of high-dimensional dataset of sensor readings in scientific measurements or in robotics. Dimensionality reduction techniques are commonly employed in all machine learning methods such as reinforcement learning [Sutton and Barto, 1998], statistical methods [Hastie et al., 2003], optimisation [Bertsekas, 1995] etc.

Dimensionality in Motor Control

In a motor control context though, the complexity of the control problem is due to the dimensionality of the dynamical system representing the plant (system under control). Thus, the reduction problem addressed in this chapter is that of finding a reduced dimensional representation of the state-space of the system in which the input-output relationship can be captured while keeping information loss to a minimum [Skogestad and Postlethwaite, 1996].

A motor task can be considered as the trajectory (time course) of a collection of one or more '*effectors*' or '*manipulanda*' (such as a finger, hand, arm eye, leg, tongue, torso, robotic limb, motor, etc.) resulting from voluntary control. Effectors are the only physical means by which an organism interact with its environment. Different combinations of effectors may be recruited for different tasks, and often a task may be fulfilled by different effector combinations. Each effector is connected via joints which allows rotation in at most 3 dimensions (eg eye, shoulder), but sometimes less (e.g. elbow, knuckle). Some effectors are mechanically linked by kinematic chains, which reduces the dimensionality of the total chain (e.g. finger-wrist-elbow-shoulder chain). Other effectors are mechanically independent and the total dimensionality remains the sum of its parts. For example, eyes are not linked mechanically, each has 3 dimensions (usually framed as horizontal, vertical, and torsional), so there are 6 dimensions in total (as seen in the chameleon).

It is crucial to recognise the difference between the mechanical dimensionality (the DoF) and the observed dimensionality. Mechanical dimensionality is the maximum physically realisable dimensionality available to the organism, but very often we observe behaviours that do not exploit this maximum. For example, unlike the chameleon, voluntary human eye movements do not have access to torsion, or change vertical disparity, so dimensionality is reduced from 6 to 3. This reduction is neural rather than a mechanical limitation.

2.2 Dimensionality Reduction in Neuroscience

Motor control and learning of motor ability is a complicated problem with many open research questions despite more than a century of research. There seem to be two broad approaches to motor control research, the viewpoint of physics / mechanics, and that of control [Latash, 2010b]. The physics approach ultimately aims at constructing a physical view of how motor control is achieved in living organisms, although it has been noted that we are far from achieving a purely physical theory of movement.

From the control perspective, the broad aim of motor control could be stated as the learning and mastering of sensorimotor transformations [Wolpert et al., 2001]; such transformations may be acquired by some kind of feedback control process, or by a feedforward process involving self-acquired models of sensorimotor relations. The learning scheme itself maybe supervised, unsupervised or reinforced by some kind of internal reward mechanism. The computational approach to motor control, is a useful framework to study the phenomenon of coordinated movement in nature [Flash and Sejnowski, 2001, Wolpert, 1997]. The aim is to identify an algorithmic representation of the coupling between the physics of the world, the anatomy, and the physiological mechanisms underlying muscles and force production along with the neural processes that lead to behaviour [Valero-Cuevas et al., 2009].

From a neuroscientific viewpoint, the role of reducing dimensionality is closely associated with the well established notion of neural mechanisms being hierarchically organised; pioneering work by the British neurologist John Hughlings Jackson in the late 19^{th} century laid the foundations [Greenblatt, 1999]. However, the control scheme itself is tightly coupled with the physics of the musculo-skeletal system and comprises many interacting elements and loops [Sherrington, 1910].

Dimensionality reduction is a recurrent theme in discussions on how the CNS seems to achieve coordination in its control of behaviour; this is the focus of a number of theories such as reflex based architectures, motor primitives etc. [Latash, 2008].

In a broader perspective, the biological theories that are related to dimensionality reduction are also inherently related to ontogenetic development of motor abilities. It has been argued that dimensionality might represent a constraint on the "learnability" of motor control [Harris, 2011], thus affecting the behaviour over the lifetime and therefore, the phenotypic fitness of an organism. Thus the Bernstein problem and the development perspective are reviewed first.

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2.2.1 Bernstein Problem and Development

The seminal research identifying motor skill learning as a problem of overcoming the hurdle of the curse of dimensionality was conducted by the Russian neuropsychologist Nikolai Bernstein [Latash, 1998], and published belatedly in the West in his work on movement coordination [Bernstein, 1967]. The key research problem he identified was on how the different degrees of freedom are harnessed to produce the movement form and variability associated with actions; this is the eponymous *Bernstein problem* or the *Degree of Freedom problem*. He viewed motor coordination as the process of mastering redundant DoF in the body and its conversion to a controllable system.

He identified the aim of movement coordination as mastering the many DoFs involved in a particular movement pattern through reduction of the number of independent variables to be controlled. Bernstein recognised that the analysis must include inertial and reactive forces along with the muscular forces since the aim of the model is not just to mime movements [Turvey et al., 1982]. The movement generation must also take into account a context-conditioned variability, i.e. in the various factors and forces that act within a given task context; thus adaptivity is essential.

Bernstein proposed that task learning is a question of increase in DoF, allowing the gradual and progressive acquisition of coordination [Bernstein, 1967]. He proposed a 3 stage model of task learning :

1. Initially, reduce DoF at periphery to minimum - a process of *freezing* DoF.
2. Gradual and progressive release of DoF restrictions - a process of *unfreezing* DoF.
3. Exploring and exploiting reactive phenomena in movement control.

His model has been tested in skill learning in humans [Vereijken et al., 1992, Newell and Van Emmerik, 1989]. Even in early development there is evidence for this mechanism in the proximodistal structure of reaching in infants [Berthier et al., 1999]. From a neural standpoint, synthetic demonstrations have established that this form of learning can result in improvements in the coordination acquired through spontaneous exploration [Ivanchenko and Jacobs, 2003,Sanger, 1994b, Sporns and Edelman, 1993].

Although his original work was proposed with DoF being defined in a biomechanical sense, a number of works have questioned the notion of DoF change accompanying task learning in a dynamical systems perspective [Schoner and Kelso, 1988]. There is a counter notion that task learning is a decrease in DoF [Mitra et al., 1998]. In this usage, the DoF coordination results in an equivalent dynamical system that resides in progressively fewer and fewer dimensions accompanying task learning [Mégrot and Bardy, 2006]. An opposing view is instead that task learning is neither a decrease or increase in DoF but instead is a change induced in constraints coupling the dynamical systems [Newell and Vaillancourt, 2001] [Berthouze and Lungarella, 2004].

Despite the controversies, Bernstein's ideas have been highly influential in contemporary theories of motor skill acquisition. It is still a high-level control oriented perspective on the motor control problem, especially concerning his theories on modular composition of movements. It is however not at all clear how these ideas can map to neuro-mechanical behaviour, and how it relates to contemporary theories for movement coordination such as those of motor primitives, forward-inverse model pairs, etc. This is partly due to the difficulty in modelling the neuromechanical apparatus accurately, as reviewed next.

2.2.2 Difficulty of Neuromechanical control

Neuromuscular behaviour is a very hard problem to model and careful attention must be paid to the physical effects that are considered, or ignored. This strongly affects the validity of any motor control theory that relies on such models [Valero-Cuevas et al., 2009]. At the physical level, vertebrate limbs are typically modelled as kinematic chains actuated by different types of muscles in agonistic, antagonistic, and multi-articulate configurations.

Muscles themselves are a critical component of physical models. The force they produce is composed of passive and active components. They produce active forces in contraction and can passively extend to accomodate external influences. Muscles are attached to the skeletal structure using tendons. Although many well known models have been utilised to explain their behaviour such as the Hill model [Hill, 1938], Voight elements etc. they are in general observed to produce a force related to their state of elongation (in length and velocity), as well as state of activation. It is however very hard to model their activation, as muscles are coupled with a very large number of motor neurons organised in the form of individual motor units [Milner-Brown et al., 1973]. Another difficulty compounding the development of neural control models are the complications and inaccuracies associated with EMG-based muscle activation estimates [Farina et al., 2004].

Furthermore, the control problem is inherently neuro-mechanical and affected by neural, mechanical as well as other factors [Mussa-Ivaldi et al., 1985]. At a neural level, dimensionality reduction is an important consequence of unsupervised learning methods based on the Hebb's Rule. It has been shown that a form of "pruning" of connections accompanies reverse hebbian learning methods [Földiak, 1990]; reverse learning based on the Oja rule has been demonstrated to result in a neuron functioning as a principal component analyser. Optimal unsupervised learning methods in nonlinear systems have been shown to result in explicit dimensionality reduction of control [Sanger, 1994a]. Developmental learning strategies have also been proposed which exploit the reduction in a neural level from such techniques [Daunicht, 1988].

In the next section the contemporary models of higher level control architectures which attempt to provide behavioural models for control, are reviewed.

2.2.3 Motor Control Architectures

Voluntary movement control in vertebrates has been associated with the cerebral cortex; the signals proceed through the spine to individual muscles. While human movement has been described as the result of optimal feedback control [Todorov and Jordan, 2002], a combined feedforward - feedback approach has been proposed to control the motion in real-time [Wolpert et al., 1998]. The combined architecture comprises of two kinds of models : inverse models in cerebellum [Kawato, 1999], which generate feedforward motor commands corresponding to a desired sensor state and forward models which predict the sensory outcome of a motor action [Wolpert et al., 1998]. Such forward models are a feature on invertebrate control as well, for instance, in insects [Webb, 2004].

Based on this control viewpoint models for motor control have been proposed that pair forward and inverse models for task learning; the MOSAIC is a significant architecture in this context [Haruno et al., 2001]. This scheme has also been proposed as a form of motor primitive inspired architecture. This is in the sense of modules which are combined to generate complex behaviours (more on primitives follows later in this section).

An alternative control scheme is that of impedance control [Hogan, 1984], where the dynamic behaviour of the limbs are modified by varying the active joint impedances through muscle co-contractions. The variation is seemingly performed in an optimal manner to stabilise the dynamics while performing tasks [Burdet et al., 2001]. This indicates that there is a tight coupling between neural control and mechanical properties of the system [Mussa-Ivaldi et al., 1994].

Although these notions are closely related to (and inspired by) control engineering models on feedback/feedforward control schemes, the role played by dimensionality comes to fore in that, the models acquired are task-specific [Tong and Flanagan, 2003]. Evidence has also been gathered for task specific control mechanisms [Braun et al., 2009] even in the case of active impedance variation [Gomi and Osu, 1998]. Task specialisation leads to a decreased number of variables specifying each individual task, which is usually a much smaller subset of the higher dimensional sensorimotor space [Wolpert and Ghahramani, 2000].

It must be kept in mind that the notion of a higher level feedforward control scheme is not the only possibility, an alternative is to exploit the natural dynamics of the musculo-skeletal structure through modulation of the multitude of control loops that are concurrently present in the body in the form of reflexes. This is reviewed next along with the equilibrium point hypothesis.

2.2.4 Reflexes and the Equilibirum Point Hypothesis

The idea of a hierarchial architecture exploiting reflexive networks dates back to British neuroscientist Sherrington's pioneering work [Sherrington, 1910]. Reflexes may be defined as high-speed low-level sensory-motor loops which can be activated by internal or external stimuli. They are tunable, not only hardwired, and Sherrington proposed that movement control is achieved by tuning parameters of the reflexes; this relies on the spring-like properties of muscles. Thus, modifying reflexes effectively modifies the body position through modulation of the equilibrium position of a set of coupled spring-like muscles acting upon the skeletal structure.

The Equilibrium Point (EP) hypothesis suggests that position control is achieved by modifying the force-length characteristic of muscles, thus affecting the angle trajectory [Feldman, 1966]. The main underlying concept is that by modification of the reflexive pathways and muscle properties shifts the system into a new equilibrium; the resultant position is a consequence of the internal and external forces acting on the system. The suggested mechanism is to modulate the strength of the tonic stretch reflex, which triggers muscle activations proportional to its elongation [Latash, 2010a].

The EP hypothesis is directly linked to dimensionality reduction since it has been proposed to apply on multi-limb systems resulting in their coordinated movement towards a kinematically specified target [Flash, 1987]; the resulting dimensionality depends on the specification of the end-point position in space. It is therefore a mechanism for redundancy resolution [Feldman, 1966].

Currently this theory is the subject of much debate and criticism [Gomi and Kawato, 1996] due to the lack of sufficient evidence [Loeb, 2010]. It has been suggested as a physics-oriented alternative to the approach using forward and inverse predictive models of control [Latash, 2010b]. Nevertheless, attempts have also been made to integrate the two views [Houk, 2010] [Shapiro, 2010].

Aside from this debate, another interesting development has been to refute the traditional notion of reflexes being innate (thus a purely phyllogenetic consequence). It has been hypothesised that reflexes developed prenatally [Marques et al., 2013] through a process of spontaneous motor activity which occurs during sleep in mammals; this proposal has been experimentally verified in examples such as rats [Robinson and Brumley, 2005] to occur in discrete multi-limb bouts [Blumberg and Lucas, 1994] [Robinson et al., 2000]. It has been hypothesised that the self-organisation of the motor circuitry is due to a reverse-hebbian learning mechanism accompanying this kind of spontaneous activity [Petersson et al., 2003]. This has lead to the idea that reflexes are a form of dimensionality reduction of the sensori-motor space resulting from pruning of neural circuitry. It has also been suggested that such a developmentally acquired set of reflexes may underlie movement coordination [Marques et al., 2013]; thus linking notions of a higher-level architecture to a self-organised reduced dimensional control strategy.

Regardless of the debate on control architectures, an alternative proposal has been to analyse some of the fundamental invariants in motor behaviour from the perspective of optimisation principles operating on the motor control; this is examined next.

2.2.5 Invariants in Behaviour - case for Optimisation

In analysis of human behaviour, a number of motion invariants are present; this is argued to be indicative of common organisational priciples underlying the computation of control. Invariants in this context refer to parameters that do not significantly change with movement speed, size, load or direction [Atkeson and Hollerbach, 1985]. Some examples of invariants in behaviour include stereotypical velocity profiles of saccadic eye movement with bell shaped velocites [Harris, 1998b], smoothness and regularities in trajectories [Richardson and Flash, 2002], and empirical laws on motion such as the Listing's law on eye rotations [Porrill et al., 2000], the Fitt's law resulting in a speed accuracy tradeoff [Harris and Wolpert, 1998], or in constraints in movement segmentation [Harris, 1998a]. A lot of research has gone into the causative factors resulting in the presence of such invariants in movement measurements over a large number and variety of subjects. As it has been mentioned earlier in this chapter, the task of motor control is ill-posed. There are nearly infinite number of solutions by which the motor system can potentially perform a task. Thus an invariant behaviour points towards a mechanism that circumvents this problem through some kind of constraint on the control.

One appealing possibility to explain such phenomena is to draw from the principles underlying Darwinian evolution in that evolution seems to maximise an organism's fitness, i.e. increase the chance of survival; in a behavioural sense this would mean optimal behaviour performance [Flash and Hogan, 1998]. The fundamental assumption is that evolution selects for overall fitness and subclasses of behaviour, such as movement will also form a part of this fitness [Harris and Wolpert, 2006]. The aim is therefore to find the relevant costs (performance criteria) and constraints which are seemingly optimised by nature; this has been the focus of significant amount of research.

Optimal control models have been proposed for arm movements which seem to optimise for smoothness [Flash and Hogans, 1985], [Hogan, 1984], torque changes [Uno et al., 1989], control effort [Daunicht, 1988], [Dean et al., 1999], variance due to noisy control [Harris and Wolpert, 1998], and for impedance compensating for the variance [Osu et al., 2004] [Burdet et al., 2001]. Such principles have also been applied to other kinds of movements such as the eyes [Harris and Wolpert, 2006] [Dean et al., 1999], optimising energy in locomotion [Anderson et al., 2001], posture [Ting, 2007] etc. It has also been hypothesised that the cost seems to contain four fundamental components - time, accuracy, stability and energy [Harris and Wolpert, 1998] with the possibility that composite costs are used [Berret et al., 2011].

An important aspect of these models that must be noted is that they do not explain how the behaviour is neurally instantiated; they only imply that certain fundamental features exist. It must be emphasised that these models do not imply that such cost functions or optimisation procedures are physically encoded in neural structures, rather, they seem to manifest during behaviour. Nevertheless, such approaches have been related to development of motor skills - it has been argued that there might additionally be a cost in how quickly an optimal behaviour can manifest during the developmental process of an organism [Harris, 2011]. Attempts have also been made to relate optimisation principles to the synthesis of primitives [Todorov et al., 2005], which are a form of reduced dimensional control [Sanger, 1994a]. The notion of primitives and modularisation of control is reviewed next.

2.2.6 Modular Control Strategies - Primitives and Synergies

The idea of modularisation is well established in motor neuroscience [Latash, 2010a]. Hypothetically, the CNS encodes a limited set of modules, or primitives, that are flexibly combined to achieve motor control. In this case dimensionality is directly related to the number of modules that are combined to generate the desired behaviour.

In general, motor primitives have been characterized as elements of computation in a sensorimotor map transforming desired limb trajectories into motor commands [Bizzi et al., 1991]. This notion has been formalized in a variety of models such as kinematic strokes, spinal force fields, muscle synergies, and central pattern generators [Flash and Hochner, 2005]. In this section we summarize the main findings on this topic, with a special emphasis on the impact of these models on dimensionality reduction.

One of the first evidence of spinal level modularity was obtained in the seminal experiments by Bizzi and colleagues, who proposed the concept of spinal force field [Mussa-Ivaldi et al., 1994]. It was shown that microstimulations of inter-neuronal regions of the spinal cord of frogs resulted in the generation of force-fields at the end-point of the limb. The obtained field depended on the region of stimulation, and it was characterized by a single equilibrium point in most of the cases. Furthermore, it was shown that simultaneous microstimulations of different regions of the spinal cord resulted in the linear superposition of the fields obtained by stimulating each region individually. These observations suggested that supraspinal regions of the CNS may elicit coordinated limb trajectories by modulating the activation of different regions of the spinal cord. The resulting trajectory would depend on the initial configuration of the limb and on the obtained force fields (that would guide the limb toward the equilibrium point). Thus, this mechanism would combine motor coordination as well as localisation of the limb in space [Tresch et al., 2002].

These experiments led to the hypothesis that spinal neural circuitries are organized into modules that realize particular motor coordination patterns, exemplified in the measured force fields. This hypothesis has been further investigated at the level of motor commands. The idea is that the hypothesized neural modules encode coordinated activations of groups of muscles, the so called muscle synergies, which can be flexibly combined to implement a desired behaviour. The notion of muscle synergy has been formalized in a variety of mathematical models, which determine different input dimensionality. The temporal [Ivanenko et al., 2004, Hart and Giszter, 2004] and the synchronous models [Tresch et al., 1999, Torres-Oviedo and Ting, 2007, Safavynia and Ting, 2012] define motor commands as the linear combination of a finite number of vectors defining the balance between muscle contractions. In the former model, specific time-courses of the weighting coefficients a_i represent the task-independent modules, and the balance vectors are the new control variables. As a result, the input dimensionality of this model (i.e. number of control variables) is equal to the number of balance vectors by the number of muscles to be controlled. The synchronous model defines the task-independent synergies as the balanced vectors, and the control input as the time-varying weights. Therefore this model, unlike the previous one, leads to a dimensionality reduction only if the number of synergies is lower then the number of muscles. The time-varying synergy model explains muscle activations as linear combinations of predefined muscle activations time-courses; each of these modules can be scaled in amplitude and shifted in time by two appropriate coefficients, which represent the new control variables [d'Avella et al., 2003, d'Avella et al., 2006, d'Avella et al., 2011]. The input dimensionality of this model is therefore equal to two by the number of synergies. It is noteworthy that the temporal as well as the time-varying synergy models allow to generate time-varying muscle activation patterns by setting the values of some time-invariant variables (i.e. balance vectors, or modulation and time-shifts). In other words, the controller does not need to "choose" appropriate motor commands at each time-step, rather it has to identify the values of the new control variables that lead to the desired time-varying control signals. As a result, such models simplify motor learning even if the number of synergies is larger than the number of variables that originally had to be controlled [Alessandro et al., 2013].

The typical approach to evaluate the hypothesis of muscle synergies consists in searching regularities in a dataset of muscle activities. Such a dataset is obtained by recording EMG signals from a group of subjects that are performing some prescribed motor tasks. Linear dimensionality reduction algorithms (e.g. Principal Component Analysis, Independent Component Analysis, Non-negative Matrix Factorization) are employed to identify a small set of components (i.e. synergies) that approximate the EMG dataset according to the chosen synergy model [Ting and Chvatal, 2010]. In many cases the extracted components appeared similar across different experimental conditions, therefore they have been regarded as an indirect evidence of an underlying modular neural organization. To establish the number of synergies that compose the hypothetical modular controller (and therefore its dimensionality), researchers typically analyse the trend of the approximation error as a function of the extracted components. The number of synergies is usually defined as the flattening point of this graph, which supposedly indicates the point that separates task-related variability by other sources of noise.

This methodology has been successful in explaining muscle contractions across a wide range of complex tasks (e.g. running, walking, keeping balance, reaching and other combined movements) in humans [Ivanenko et al., 2005, Cappellini et al., 2006, d'Avella et al., 2008, d'Avella et al., 2006, d'Avella et al., 2011, Torres-Oviedo and Ting, 2007, Torres-Oviedo and Ting, 2010], in frogs [d'Avella and Bizzi, 2005, Giszter et al., 1993, Mussa-Ivaldi et al., 1994, Mussa-Ivaldi and Bizzi, 2000, Kargo and Giszter, 2000, Kargo and Giszter, 2008], cats [Ting and Macpherson, 2005, Torres-Oviedo et al., 2006] and monkeys [Overduin et al., 2008, Overduin et al., 2012].

In order to evaluate whether these results effectively reflect a modular neural organization, other researchers attempted to relate neural activity with simultaneously recorded muscle activation during performance of different motor tasks. Such tests were successful in many cases, suggesting that neural populations constitute neural bases for synergistic muscle contractions [Hart and Giszter, 2010, Holdefer and Miller, 2002, Overduin et al., 2012]. However, the regions of the CNS where muscle synergies might be implemented is still not clear; some studies suggest the spinal cord [Hart and Giszter, 2010], while other indicate different regions of the CNS [Thoroughman and Shadmehr, 2000, Cheung et al., 2009]. The reader is referred to [Alessandro et al., 2013] for a review on muscle synergies.

There are arguments against the hypothesis of muscle synergies [Tresch and Jarc, 2009] due to their phenomenological nature [Todorov and Ghahramani, 2003], and many challenges remain open [Kutch and Valero-Cuevas, 2012, Valero-Cuevas et al., 2009]. Nevertheless, the relevance of motor primitives have been emphasised from a dimensionality perspective [Ting, 2007], as well as from that of optimisation and learning [Todorov et al., 2005]; i.e. a hierarchical structure where the low-level consists of appropriate primitives aids learning by ameliorating the curse of dimensionality. Primitives have also been related to development, with evidence pointing to their presence in infants [Dominici et al., 2011]. The aspect of dimensionality reduction is particularly emphasized in the work by Berniker and colleagues, who proposed a computational method to synthesize synergies based on model dimensionality reduction of the dynamics [Berniker et al., 2009, Berniker, 2005]. This approach is explained further in section 2.4.

A closely related notion of modular control is that of Central Pattern Generator (CPG), which have been shown to underlie vertebrate locomotion [Ijspeert, 2008]. CPG research focuses on identifying neural structures that can output rhythmic patterns under aperiodic stimulation; the reduced dimensionality is in the space of the modulating signals. This concept is further detailed elsewhere [Ijspeert, 2008]; a key demonstration has been the identification of signals that lead to gait transitions in a variety of vertebrates, for example walking to swimming in the salamander [Ijspeert et al., 2007].

Although most of the experimental research has focused on human and primate subjects, the proposal for primitives has also been extended to diverse morphologies such as invertebrates like insects and cephalopods [Flash and Hochner, 2005]. One such hypothesis is for the existence of a peripheral motor program in the octopus arm control [Sumbre et al., 2001], evidenced by patterns of muscle activation underlying reaching movements [Gutfreund et al., 1998]. The dimensionality reduction on an octopus arm, which is mechanically an infinite dimensional structure, results in as low as 2 parameters being needed to specify reaching behaviours through combinations of the activation patterns [Yekutieli et al., 2005a, Yekutieli et al., 2005b].

Inspired by a number of these ideas, the field of robotics has started to consider control techniques based on reduced dimensional principles and biomimetics. This is surveyed in the next section.

2.3 Dimensionality Reduction in AI and Robotics

In the field of robotics, the effects of high-dimensionality are well known from two important perspectives, learning and optimisation (see Sec. 2.1.2), and redundancy in control.

2.3.1 Dimensionality in GOFAI and Embodied AI

Traditional control architectures for robots tend to be very centralised and involve symbolic manipulation in the high-level which is translated to motor commands - a paradigm referred to as Good Old Fashioned AI (GOFAI) [Pfeifer and Scheier, 2001]. Many such schemes are hierarchical and organised in layers [Prescott et al., 1998], incorporating low-level feedback control mechanisms and high-level feedforward mechanisms which may operate on kinematic or dynamic descriptions of desired behaviour [Spong and Vidyasagar, 2008]. In the GOFAI paradigm, there existed a clear distinction between perception, planning and control mechanisms with interaction between the three components specified usually in an external frame of reference, such as in a Cartesian task and trajectory specification [Siciliano and Khatib, 2008]. Within this framework the prohibitive cost of high-dimensionality on movement generation is well known and heavy effort must be invested in the control design process for the modelling and system identification [Featherstone, 2007]. One approach has been to instead solve the problem of large kinematic redundancy using pseudo-inverse methods on a robot kinematic Jacobian [Hollerbach and Suh, 1987].

An alternate philosophy that has emerged in recent years is that of *embodiment* [Pfeifer and Scheier, 2001]. Based on self-organisation principles and strongly inspired by biological models, robotics has faced a paradigm shift in the techniques enabling manifestation of intelligent behaviour [Pfeifer et al., 2007]. Following this philosophy, architectures based on the principle of parallel loosely coupled processes [Pfeifer and Bongard, 2007] such as the subsumption architecture [Brooks, 1986] have pointed towards representation-free forms of intelligence [Brooks, 1991].

Although reduced dimensional concepts seem irrelevant to such mechanisms at first sight, approaches such as behaviour-based robotics have striven to find a common ground between a representation free architecture and a model-based hierarchical approach by taking inspiration from nature [Matarić, 1998]. However in terms of morphology and material properties, bio-inspiration and biomimetics have lead to research on under-actuated and soft robots, which have brought new perspectives to the dimensionality issue.

2.3.2 Bio-Inspired and Biomimetcs

In addressing the question of how to make machines as adaptive as living creatures, one proposal has been to directly take inspiration from nature in the design of the morphology and material properties [Pfeifer et al., 2007]; this has also necessitated research in bio-inspired intelligence [Floreano and Mattiussi, 2008]. One such approach, which directly relates to the dimensionality issue is that of under-actuation.

Under-actuated robots are so called, because they possess fewer actuated DoF than the net mechanical DoF [Tedrake, 2009]. The principles by which they are controlled involve exploitation of the natural dynamics of the mechanical system; often employing compliance at joints etc.

Often such robots are unstable in their dynamics and the control schemes incorporate innovative methods to render them controllable, for eg. exploiting the resonances through dynamic oscillators [Righetti et al., 2006]. Under-actuation methods are starting to find many application in problems which are traditionally hard to solve such as walking [Collins et al., 2005], grasping [Bicchi and Kumar, 2000]and manipulation [Birglen et al., 2008] etc. Despite the high performance demonstrated in energy efficiency or in dynamic behaviour adapting to unknown perturbations such as uneven terrain, so far this approach has yielded machines with only a limited scope of behaviour. Nevertheless, the dimensionality reduction applies to the number of input signals that must be computed in real time as an alternative to traditional robotics.

Another recent morphological advancement has been driven by biomimesis through exploitation of new materials. Mechanisms like the series elastic actuator and advancements in muscle like elements such as the McKibben mechanism, and electro-active polymers etc. have realised the prospect of building machines that mimic biological structures closely [Bar-Cohen, 2005]. This has lead to development of anthropomimetic machines which have a potential to serve as test platforms for neuroscientific concepts [Wittmeier et al., 2013]. Diverse morphologies can be tackled with these technologies such as effective designs for hyper-redundant or continuum robots [Trivedi et al., 2008]. In the case of humanoids, artificial muscles allow force production using antagonistic pairs of actuators; the mechanical properties of the actuators infuse useful passive properties in the joint level that can be modulated for tasks such as throwing, or complex manipulation [Braun et al., 2012].

Nevertheless, in such morphologies the complications in control due to dimensionality is obvious; formulations of dynamics are complex for both compliant robots [Luca and Book, 2008] and continuum robots [Webster and Jones, 2010]. Reduced dynamic modelling techniques could allow an efficient exploitation of the mechanical properties in morphologies with passive compliance [Seyfarth et al., 2009]. Another approach might be to use unsupervised learning methods to acquire low-level reflexes in tendon driven structures which may then be coordinated in a higher level architecture [Marques et al., 2012].

Despite the many advantages introduced by such approaches, modelling and control is a complex process and requires a great deal of creativity on the part of the designer; looking at nature to understand reduced dimensional control principles might be a way out. One possible idea is to take inspiration from the natural development of skills in organisms; thus allowing the robot to self-learn its behaviour through interaction with the environment - the approach of developmental robotics.

2.3.3 Developmental Robotics

The goal of developmental robotics is to understand the biological ontogenetic development of skills and implement them in robots [Asada et al., 2001]. Intelligent behaviour is thus hypothesised to emerge through a process of self-learning through interaction with the environment. Different kinds of skills can be acquired by such methods; spontaneous agent-centered acquisition of motor ability is the focus of motor control development research [Lungarella et al., 2003, Lungarella, 2007].

The dimensionality of the organism is relevant in the self-acquisition of a body-schema, an important cognitive concept interlinking perception and action [Hoffmann et al., 2010]. One of the proposals for bootstrapping the motor control learning is known as motor babbling [Saegusa et al., 2009]. It has been noted that the related self-organised unsupervised learning of the sensori-motor map can result in the learning of coordination through a technique of goal babbling instead [Rolf et al., 2010], i.e. utilising the lower dimensionality of the task-space. This framework has also been employed in the notion of a playful machine which can cope with large body DoF through self-exploration [Der and Martius, 2012]. Spontaneously acquired sensori-motor coordination relates directly to dimensionality reduction methods [Te Boekhorst et al., 2003]. Furthermore, sensori-motor reduction as a natural process resulting from spontaneous activity in sleep can inspire novel developmental methods for robots [Blumberg et al., 2013].

An important outcome of one such developmental model, which is inspired by the Bernstein approach, is the proposal to bootstrap sensori-motor space exploration through fewer degrees of freedom; this can be followed by progressive unfreezing in later stages [Lungarella and Berthouze, 2002]. Such an approach can also be expanded to include alternate stages of freezing and freeing of DoF. This allows a gradual increase in task complexity that can be tackled by the robot [Berthouze and Lungarella, 2004]. This approach, while promising is yet to be tested on high-dimensional systems, and it is the view of this thesis that grounding the framework on a sound theoretical basis might be the way forward.

A direct approach to reducing dimensionality at the input is to implement robot control architectures inspired by the modularity of biological motor primitives as discussed next.

2.3.4 Motor Primitives and Synergies in Robotics

It could be argued that the idea of modularity is not very new in robotics, and approaches such as the subsumption architecture have dealt with it in a layered manner [Brooks, 1986]. However, more recent robot control schemes have directly taken inspiration from the notions of motor primitives as defined in biology [Konczak, 2005]. In particular, the concept of muscle synergies has led to novel control architectures that generate actuations as a weighted summation of a finite number of predefined control signals (i.e. motor synergies). Such a scheme reduces the number of variables to be controlled (synchronous synergy model), or more generically, the dimensionality of the actuation signals (time-varying synergy model). Thus, it reduces the time required for motor learning [Chhabra and Jacobs, 2006, Todorov and Ghahramani, 2003].

Restricting the admissible actuations to linear combinations of synergies limits the control signals that can be generated, and therefore the tasks that can be executed. Thus, the main challenge becomes the synthesis of a minimal set of synergies that allows the accomplishments of the desired tasks. Nori and Frezza have proposed a closed-form solution for synergies that preserve the controllability of a feedback-linearised system [Nori and Frezza, 2005]. Other studies have shown that the actuations underlying the execution of movements are effective synergies to achieve similar motor tasks [Chhabra and Jacobs, 2008]. In particular, the Dynamic Response Decomposition (DRD) offers a computationally fast method to synthesise such primitives, and provides a mathematical framework to generate synergy-based controllers [Alessandro et al., 2012]. These studies have been tested on simulated kinematic chains performing point-to-point and via-point reaching tasks.

If the dynamical model of the robot is unknown, motor synergies can be synthesized by means of computationally intensive algorithms based on optimisation and learning. In some cases such procedures have been used to identify the synergies that were better suited for a particular set of training tasks [Alessandro and Nori, 2012, Thomas and Barto, 2012]; otherwise, unsupervised learning strategies have been employed to identify primitives that reflected the biomechanical constraints of the robot [Marques et al., 2012, Todorov and Ghahramani, 2003].

It is worth mentioning a line of research that exploits the concept of synergies to simplify the control of robotics hands. Malhotra and colleagues identified synergies for a tendon-driven robotic hand by applying PCA to a dataset of tendon-lengths [Malhotra et al., 2012]. Other researchers defined synergies at the kinematic level, rather than at a actuation level. In particular, inspired by previous human studies [Santello et al., 1998], they restricted the admissible joint-postures of a robotic hand to linear superpositions of a small number of kinematic components, called eigengrasps [Bicchi et al., 2011]. Such components are taken from human experiments and adapted to the robot mechanical structure. Other researchers have proposed a direct mechanical implementation of the eigengrasps [Brown and Asada, 2007]. Finally, the idea of kinematic synergies has been used for the whole body balancing of a humanoid robot [Hauser et al., 2007, Hauser et al., 2011]; in this study the authors propose a method to construct kinematic synergies (i.e. predefined balance between joint positions) that allow a linear mapping between synergy-weights and task variables.

2.3.5 Dynamical Movement Primitives

In recent years, the dynamical systems approach has found applications in robotics; this is inspired by neuroscientific models based on attractor systems [Ijspeert, 2008] [Schoner and Kelso, 1988]. In this context, Dynamical Movement Primitives (DMP) were proposed [Schaal et al., 2003, Schaal et al., 2005] as a planning and control architecture that employs the notion of tunable attractor landscapes [Ijspeert et al., 2003].

DMPs are learnable nonlinear dynamical systems which encode trajectories [Schaal et al., 2007, Ijspeert et al., 2003]. They allow movement plans to be encoded and reproduced with a set of parameters, that can be learned using regression based methods [Schaal and Atkeson, 1998]. The architecture consists of controllers based on tunable nonlinear dynamical systems, and can be programmed to learn complex, discrete or rhythmic, movements from a training trajectory. The controllers can be considered to be discrete or rhythmic pattern generators which can replay and modulate the learned movements, while being robust against perturbations. Imitation learning is thus demonstrated as a viable proposal by which complex morphologies can achieve coordinated behaviour [Schaal et al., 2003], [Nakanishi et al., 2004] [Ijspeert et al., 2001].

The principle feature that makes them an attractive choice for encoding motor primitives in artificial systems, is that the planning of tasks is carried out in a linear space. They have been proposed as a framework unifying the seemingly disparate approaches of dynamical systems and optimisation for the learning of motor control [Schaal et al., 2007].

The DMP framework has found applications in a wide variety of robotics problems such as humanoid control [Ijspeert et al., 2013], quadrupeds, and flight control [Perk and Slotine, 2006]. DMPs capture the complexity of the motor coordination problem through translation into a task independent basis space. Dependent on trajectory complexity, weights may also be learned online [Gams et al., 2009]. Some novel applications of the framework include learning impedance control in a humanoid [Buchli et al., 2011] and modelling the convergent force fields of frog leg wiping behaviours [Hoffmann et al., 2008]. DMPs thus are a promising framework to implement primitives in complex robots. Interestingly, recent works have also started to examine the improvements brought forth by incorporating dimensionality reduction techniques in their learning [Bitzer and Vijayakumar, 2009], [Bitzer, 2011].

Although robotics and AI has significantly benefited from inspiration from biology, in our survey we take the view that the dimensionality issue is only dealt with tangentially in most works; i.e. from the perspective of self-organisation or bio-inspired control innovations. There is clearly a need for a firm theoretical basis in this problem; methods based on control theoretic viewpoint which are reviewed next, may prove to be the solution.

2.4 Dimensionality Reduction in Control Engineering

The difficulties of controlling complex systems are well known in control engineering [Skogestad and Postlethwaite, 1996]; dimensionality remains an extremely difficult phenomenon to deal with. With recent advancements in computer hardware and in numerical methods, simulation has become an effective way at understanding the behaviour of such systems. However complexities in the simulation can arise from a large dimensionality due to the need to store and compute a huge number of variables. Often physics-based representations of a system tend to be verbose in terms of number of state variables. Not all of these variables are necessarily relevant to the control problem being simulated, and may complicate the problem numerically, apart from not even being measureable in real-time. This has motivated research into tools and techniques which can compute a dimensionality reduction; the reduced order model synthesised may be used suitably for simulation, computing control and behaviour prediction [Antoulas et al., 2001] as depicted in Fig. 1.1 in Chap. 1.

2.4.1 Model Order Reduction (MOR)

The algorithms of Model Order Reduction (MOR) are closely related to system identification [Ljung, 1999]. In particular MOR develops on work on minimal realisation of systems by Hungarian-American engineer, Rudolf Kálmán, aimed at deriving the most compact representation of a system behaviour in terms of order [Schutter, 2000]. Reduction techniques are utilised in subspace methods which basically utilise a high-dimensional system initially to represent a data and then prune the state-space until it is minimum in dimensions, while also satisfying some quality measure. The standard approach is therefore to determine some optimisation criterion and a noise model which fits the data (signals) from the system [Van Der Veen et al., 1993].

The most widely used MOR framework is that of projection. Consider a generic dynamical system given by,

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \tag{2.5}$$

where the system is of dimensionality $\mathbf{x} \in \mathbb{R}^N$, and input $\mathbf{u} \in \mathbb{R}^I$, then the aim is to find a reduced dimensional representation by projecting the state variable into a subspace through the projection operator P,

$$\mathbf{z} = P\mathbf{x}, \tag{2.6}$$

where $\mathbf{z}$ is the new state variable of dimensionality k such that $k << N$. The new dynamical system is then as,

$$\dot{z} = \hat{f}(z, u), \tag{2.7}$$

this process usually employ a galerkin projection [Skogestad and Postlethwaite, 1996]. Note that if the system is modelled by some form of Partial Differential Equation (PDE) instead of the representation as a set of first-order ODEs, then a process of discretisation [Hahn and Edgar, 2002b] must be employed first to obtain a system of the form in Eq. 2.5.

There are a variety of Linear MOR techniques algorithms based on projection (reviews on the topic may be found in [Antoulas et al., 2001] [Skogestad and Postlethwaite, 1996]). The methods of proper orthogonal decomposition and balanced truncation are summarised here. While the former is better in understanding the role played by the dynamics of a system in a reduction, the latter has applications for control problems. These two approaches are depicted in Fig. 3.2 (Chap. 3).

2.4.2 Proper Orthogonal Decomposition (POD)

POD is a multi-variate statistical method that obtains a compact representation of a dynamical system by projecting it into a lower dimensional subspace; the aim is to retain as much variation as possible in the reduced dimensional space [Kerschen et al., 2005]. The POD method in a finite dimensional case is the same as principal component analysis (PCA), a popular technique for data dimensionality reduction. POD thus projects the dynamics into a subspace capturing the most "*statistical energy*" [Kerschen et al., 2005]. POD has been applied to a variety of model reduction problems [Antoulas et al., 2001] and stability preserving variations have been developed for control applications [Prajna, 2003].

POD is very useful in structural dynamics for modal analysis. It has been demonstrated that the proper orthogonal modes of some kinds of linear systems are identical to the normal modes (vibration eigenmodes) [Kerschen and Golinval, 2002]. Identification of mode shapes is essential to understand how complex structures react to applied forces; applications include the analysis of stability of bridges etc. This method has also been applied to active control of flexible structures [Al-Dmour and Mohammad, 2002].

However, POD-based methods are inherently data-driven, i.e. utilise datasets of system behaviour elicited using standard perturbations; it is sometimes more beneficial to directly generate a reduction for a known system without simulating/testing its behaviour first. Furthermore, POD-based methods are known as input-state methods since they compute a reduction using a state trajectory generated with a standard known input. In many problems, a distinction is made between the state-space and output space of a system and control needs to compute a desired output trajectory. Thus a method that computes input-output reduction is preferable for control problems [Lall and Marsden, 2002].

2.4.3 Balanced Reduction

Balanced model reduction methods extended the minimum realisation theory of Kalman to account for the controllability and observability of systems using a principal component analysis [Moore, 1981]. In many control problems it is sufficient to suitably model the input-output behaviour and this is best captured by the system controllability and observability gramians. Furthermore, model reduction in this paradigm can also provide insight on the causes underlying the observable dynamics of a system [Hahn and Edgar, 2002b].

The method of balanced reduction first computes a rotation of system coordinates in order to '*balance*' the observability and controllability gramians; the states which have the greatest contribution to the input-output behaviour are obtained. Performing a Galerkin projection on the most important of these state variables yields a very effective model reduction that is ideally suited for control [Hahn and Edgar, 2002a]. One measure to quantify this importance is known as the Hankel Singular Value (HSV). It is computed as the square root of the eigenvalues of the product of the controllability and observability matrices, giving a quantitative 'score' to the importance of each of the balanced state variables. They can thus be examined to determine the subspace to which the system must be reduced to.

Balanced reduction methods maybe computed using the SVD and are numerically efficient for large scale systems [Mehrmann and Stykel, 2005]. While the necessary and sufficient conditions for the presence of such a reduction have been examined [Kenney and Hewer, 1987], variations have been developed for time varying systems as well [Sandberg and Rantzer, 2004].

There are broadly two variations to applying balanced reduction methods, the first is simply truncation - the balanced system is simply reduced to a subset of state variables. The second approach known as the singular perturbation approximation [Liu and Anderson, 1989] sets the variables to their steady state values. While the former is better at retaining the frequency response behaviour of the source system, the latter captures steady state behaviour better; the appropriate method for a given problem can thus be chosen. Balanced reduction methods are also related to POD and combined approaches have been proposed to capture the advantages of both methods since POD is numerically simpler to compute for high-dimensional problems [Willcox and Peraire, 2002].

2.4.4 Methods for Nonlinear Systems

Although the current state-of-art in theory for linear model reduction techniques is extensive, in the case of nonlinear systems a significant theoretical basis is currently lacking [Hahn and Edgar, 2002b]. POD-based methods have been applied to nonlinear systems [Moore, 1979] although their generalisation is limited. The primary problem is that a statistically computed linearly projected subspace may not capture the input-output dynamic behaviour of the source nonlinear dynamical system to the best extent possible. One effect that has been observed in the application of POD to nonlinear systems is a sensitive dependence on the input energies at which the POD results are synthesised [Kerschen and Golinval, 2002].

Some linear approximated approaches include, local linearations of the source system followed by linear balancing, and a technique of trajectory piecewise model reduction [Rewienski, 2003]. An alternative is known as the method of energy functions and develops a covariance matrix based reduction to the system, although this is computationally intensive [Scherpen, 1993].

However, one family of methods that is promising for nonlinear applications is that of empirical gramians [Lall and Marsden, 2002]. These methods combine some ideas from POD in synthesising gramians through a data-driven approach, by supplying the system with impulsive perturbations. The empirical gramians are measures that are equivalent to the standard controllability and observability gramians for the linear case, but suitably approximate them for some region of state-space (defined for inputs within some bound of energies). The empirical gramian approach has been used to synthesise a model reduction technique [Hahn and Edgar, 2002a] that has found successful application on some nonlinear model reduction problems.

As a final note, model reduction techniques have so far not been used substantially in robotics although there is potential in some problems such as contact dynamics simulation [Ma, 2004], and in synthesis of synergies for manipulators [Malhotra et al., 2012]. Lastly, the empirical gramian approach has even been used for biological primitive synthesis models [Berniker, 2005]; it is used within a primitive computation algorithm to reduce the dimensionality of a model of the frog leg with many nonlinear muscles attached.

2.5 Summary

This chapter presented a broad overview of dimensionality reduction for motor control from three perspectives. In neuroscence, high-dimensionality and the difficulty in task learning and control are directly dealt with by Bernstein, who proposed a three stage learning model to cope the difficulties of a large number of bio-mechanical DoF. There is an active debate however, if the notion of dimensional change applies in a biomechanical sense as he proposed, or in a dynamical sense. In a real-time control perspective, a number of architectures address the question of how to generate signals coordinating the muscles for performing tasks; the complexity of neuromechanical modelling was summarised. Two approaches for control architectures were presented, one from a control theory inspired notion of paired forward and inverse models regulating behaviour, and a hierarchical control of parameters in a network of reflexes; the equilibrium-point movement arising from natural biomechanics. There are attempts to establish a common ground between these theories, and the relevance to dimensionality is that somehow the task space dimensionality seems to dictate the behaviour and coordination.

It has been suggested that the space of possible solutions (and maybe its dimensionality as well) is constrained due to evolutionary requirements; hypotheses of optimal motor control imply that redundancy resolution arises due to optimisation of some form of performance index. Alternately, it has been proposed that the space of inputs is constrained by modularisation in the form of motor primitives. The convergent force-field and the muscle synergy hypothesis are archetypes of this notion where the space of inputs is constrained into a linear combination of patterns. The dimensionality problem and a mitigation strategy remains a consistent undercurrent in all of these theories and there is definite value in seeking inspiration from nature for the design of embodied systems.

In robotics and AI dimensionality reduction has so far been tackled mostly in an implicit manner. In classical hierarchical approaches to robot control in the form of the pseudo-inverse of the Jacobian, the cost of computation of forward and inverse dynamical solutions for control are well known. In AI and machine learning, the difficulties of dealing with high-dimensional spaces are well understood, as captured by the aphorism, the "*curse of dimensionality*" . Dimensionality reduction is seen as one way to cope with these difficulties.

In recent times, the philosophy of embodiment has resulted in a paradigm shift in the traditional ideas of robot architectures and learning. One of the consequences has been that of novel morphologies and materials in robot bodies driven by bio-inspiration. Two seemingly opposing approaches are of underactuated robotics and biomimetics. Dimensionality reduction is tackled in underactuated robotics through minimisation of the number of actuation signals that are computed in real-time; control is achieved through exploitation of natural dynamics. The converse approach is to utilise biomimetic principles and incorporate a large number of muscle-like actuators to provide the necessary forces for motor behaviour; this leads to an increase in dimensionality of signals computed in real-time. In this case, the need for dimensionality reduction in the control strategy clearly comes to the fore.

One strategy to acquire motor coordination in such high-dimensional morphologies has been that of self-acquisition of abilities and interaction with the environment. This approach of developmental robotics has also proven useful for biological modelling; dimensionality reduction has been suggested as emerging as a consequence of the synthetic ontogenetic learning process. In a real-time control perspective, neuro-inspired control strategies have also been proposed for achieved motor coordination in high-dimensional robots; architectures inspired by motor primitives and muscle synergies have been demonstrated. The implementation of the Dynamical Movement Primitives (DMP) is noteworthy in this respect. In summary, there is clearly a need for explicitly coping with the high-dimensionality control problem. Posing the problem using a sound theoretical basis might yield effective strategies for suitably exploiting reduced dimensional principles in robot designs and controllers.

Lastly, in the control theory perspective, dimensionality reduction has aimed at simplifying some of the engineering process, i.e. that of simulation, modelling and control. The aim of model order reduction has been to develop tools and techniques for synthesising reduced order representations of systems capturing the dynamical behaviour to the best extent possible. The resulting simplified models may be employed for simulation and control of complex systems. The so-called projection framework is a useful method for order reduction. Proper orthogonal decomposition, a statistical technique, is useful in many kinds of applications due to its relationship with methods for modal analysis of systems. However if the aim is to capture the input-output behaviour of a system in the best extent possible, balanced reduction methods are preferable.These methods have also suitably been extended for nonlinear applications by using empirical measures to characterise the reduced dimensional behaviour. These methods have proven to be useful in the reduction and control of many kinds of complex systems, and clearly present an opportunity for exploitation in robotics and biological research.

Chapter 3

Mathematical Framework and Summary of Studies

In this chapter the mathematical framework of reduced dimensionality analysis is introduced and a summary of the main studies carried out is presented. As stated in Chap. 1, the aim of this thesis is to investigate the research question :

What is reduced dimensionality and how can it be exploited in the design and control of embodied systems?

Towards realising this aim, a quantification of the notion of reduced dimensionality is sought. In the literature survey presented in Chap. 2, the need for a consistent and sound theoretical framework for understanding reduced dimensionality was emphasised. To this end, a consistent mathematical framework of *reduced dimensionality analysis* was developed by utilising a control theoretic perspective of dimensionality. Subsequently, this framework was utilised for a systematic exploration of the factors that influence reduced dimensionality in the design and control of a system. The five explorations are conceptually depicted in Fig. 3.1.

From a control-theoretic viewpoint of a systems behaviour, the state-space formulation is often a convenient way to represent both linear and nonlinear dynamics. Hence this view of the system behaviour is taken as the basis of the analysis framework. The state-space control representation also naturally motivates the study of the reduced dimensionality problem from the three perspectives of (i) input (Chap. 6) , (ii) state dynamics (Chap. 4, and (ii) output (Chap. 5). Two additional case studies examine the relevance of the extracted design principles from the perspectives of robot control using Dynamical Movement Primitives (Chap. 7) and a developmental approach for motor control through dimensional change (Chap. 8.

The reduced dimensionality analysis framework is presented next in Sec. 3.1 and used to provide a consistent mathematical vocabulary for summarising the main contributions in the subsequent sections.

3.1 Reduced dimensionality analysis

The notion of reduced dimensionality was defined broadly in Sec. 1.2 as the phenomenon of a higher dimensional system naturally facilitating the effective reductions of its own dimensionality (the effectiveness depending on its intended behaviours). In this thesis, this notion is quantified

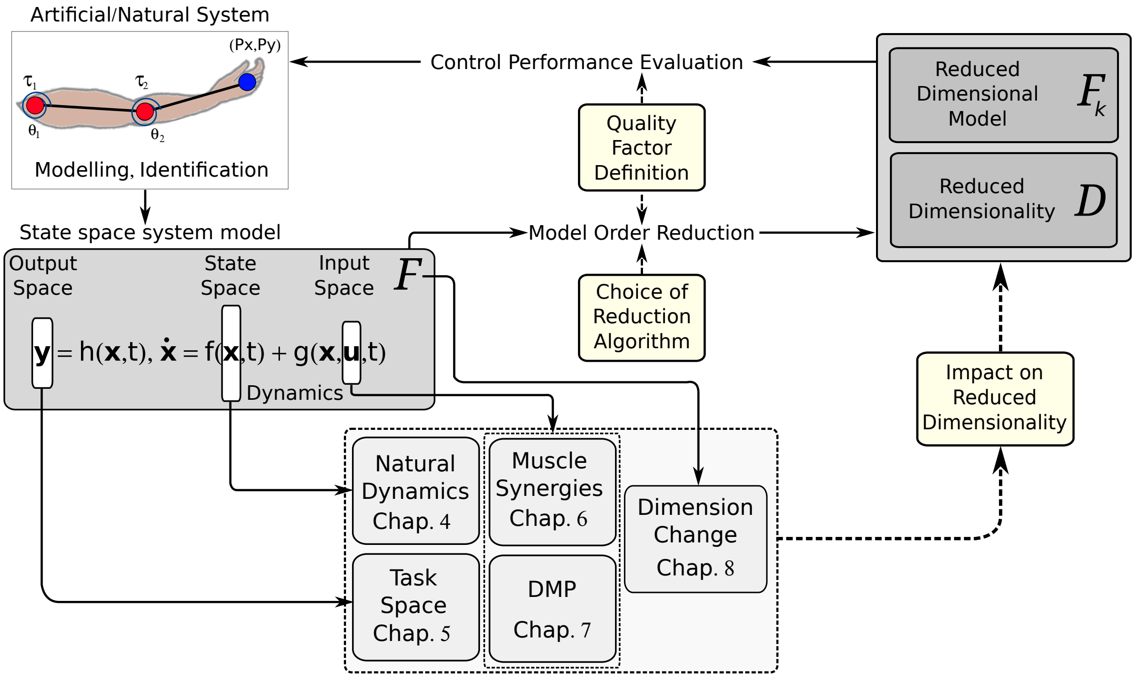


Figure 3.1: Mathematical framework of reduced dimensionality analysis and the studies carried out in this thesis

from a control-theoretic perspective since the expected outcome is a hypothesis for the control of an embodied system. Moreover, as described in Sec. 2.4, control theory naturally affords a useful vocabulary for describing the reduction in dimensionality of a dynamical system due to the theories of minimum realisation and system identification.

3.1.1 Dimensionality Reduction Problem

The problem of dimensionality reduction of a dynamical system can be posed as follows. Consider a system described by the following dynamics using a set of ODEs,

$$\mathbf{y} = h(\mathbf{x}, t), \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, \mathbf{u}, t), \tag{3.1}$$

where the variables $\mathbf{x}(t)$ denotes the state, $\mathbf{u}(t)$ the input, and $\mathbf{y}(t)$ the output. For this system the dimensionality can be described by $\mathbf{x}(t) \in \mathbb{R}^N$, an N dimensional state space, and the inputs lie in $\mathbf{u}(t) \in \mathbb{R}^{N_i}$ and outputs $\mathbf{y}(t) \in \mathbb{R}^{N_o}$; Note that, N_i and N_o need not be equal to N. Let us denote this system by the *functional* $\mathcal{F}(f(.), g(.), h(u)$. This form of representing the dynamics allows the separation of analysis of the so-called natural dynamics represented by $h(\mathbf{x}, t)$, from the input dynamics represented by $g(\mathbf{x}, \mathbf{u}, t)$ and the output map represented by $h(\mathbf{x}, t)$. In the case of robot dynamics, the input dynamics are affected by the actuation used, the passive mechanical properties affect the natural dynamics and choice of the intended task defines the output.

For the system in Eq.(3.1), an equivalent reduced dimensional dynamical representation is then of the form,

$$\tilde{\mathbf{y}} = \hat{h}(\mathbf{z}, t), \quad \dot{\mathbf{z}} = \hat{f}(\mathbf{z}, t) + \hat{g}(\mathbf{z}, \mathbf{u}, t), \tag{3.2}$$

such that the new dimension reduced state variable is denoted by $\mathbf{z}(t)$ and it lies in the space $\mathbf{z}(t) \in \mathbb{R}^k$, where $k \leq N$. This new system, is then denoted by $\mathcal{F}_k(\hat{f}, \hat{g}, \hat{h})$ and is a reduced dimensional representation of the system $\mathcal{F}$ in Eq. (3.1) since it captures the input-output dynamic relationship using a reduced dimensional state space.

The general dimensionality reduction problem from a control-theoretic perspective is thus a question of synthesising the system in Eq. (3.2) from the original system such that k is minimum for some measure of quality of reduction. Thus, the definition of this measure of quality is the quantification of reduced dimensionality that is sought.

3.1.2 Quantifying Reduced Dimensionality

The system in Eq. (3.2) represents only a representation of the original system. Note that for the same input signal $\mathbf{u}(t)$, the loss of information in the model in Eq. (3.2) results in a deviation in the output of $\tilde{\mathbf{y}}(t)$. The main observation of minimal realisation theory is relevant in this case, if there was no difference in the input-output behaviour, that implies that the reduced dimensional system of Eq. (3.2) simply is a more compact representation of the original system; i.e. the first system is over-defined in the sense of its state space.

In this thesis, it can be assumed that the original system is minimal in realisation itself and that the reduced dimensional system derived by some method results in a measurable difference in input-output behaviour that is bounded by a reduction error factor ϵ which can then be is used in a measure of quality of dimensionality reduction $\mathcal{Q}$ as,

$$\mathcal{Q}(\mathcal{F}_k) - \mathcal{Q}(\mathcal{F}) \leq \epsilon, \tag{3.3}$$

This measure can practically be defined on the basis of ability to replicate some benchmark input-output relationship $\{\mathbf{y}(t), \mathbf{u}(t)\}$ (i.e. useful behaviour that is desired). For instance, for a benchmark input signal $\mathbf{u}(t)$, the quality factor can be defined as $\|\tilde{\mathbf{y}}(t) - \mathbf{y}(t)\|_2$. This measure can then be used to classify a *useful* reduction i.e. a reduction which can be useful for the purpose of control synthesis and behaviour analysis.

This definition is important in ensuring that the reduction does not excessively impact the intended behaviour. In the scope of this thesis, since an aim of the reduction is to synthesise control and observer strategies, the quality measure is defined on the comparison of the output trajectories of the full and reduced dimensional system for some standard benchmark perturbations. The measure quantifying reduced dimensionality that is sought can thus be defined by,

$$\mathcal{D}(\mathcal{F}) = k, \tag{3.4}$$

such that $k, N \in \mathbb{Z}^+$, the set of integers, and $1 \leq k \leq N$. Also, by definition, for any such reduced dimensional measure k, there exists a corresponding reduced dimensional representation given by Eq. (3.2), i.e. the measure k thus captures the complexity of a reduced dimensional representation of a system. An important feature of this measure that must be noted is that it is an *integer* measuring of reduced dimensionality. In the subsequent sections of this chapter, the contributions are presented in the perspective of this mathematical description.

The research presented in this thesis utilises two kinds of algorithms for the quantification of reduced dimensionality by the quality factor Eq. (3.3). These are : (i) Proper Orthogonal Decomposition (POD) and (ii) Balanced Reduction (BR) (see Sec. 2.4.1). The two approaches are depicted in Fig. 3.2. Standard signals are used to perturb the signal and obtain datasets of state and output trajectories. POD is a input-state method and thus utilizes the signals $\mathbf{u}(t)$, and $\mathbf{x}(t)$. BR instead uses the signals $\mathbf{u}(t)$ and $\mathbf{y}(t)$ since it is a input-output method. In a linear dynamical system case however, balanced reduction methods can be computed by an analytical formulation independent of the actual signals.

In the case of POD, the measure of Eq. (3.4) is computed using thresholded normalised Proper Orthogonal mode Magnitudes (POM), and in balanced reduction, it is using Hankel Singular

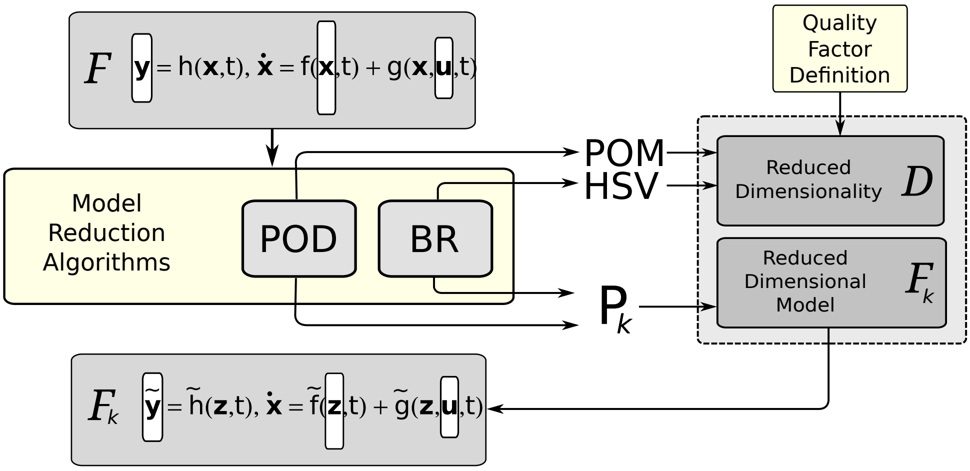


Figure 3.2: Reduced dimensionality analysis and methods used in this thesis : (i) Proper Orthogonal Decomposition (POD) and (ii) Balanced Reduction (BR). The N dimensional system $\mathcal{F}$ is reduced to a k dimensional system $\mathcal{F}_k$. The reduced dimensionality $\mathcal{D}$ is derived by using a quality threshold on the corresponding measures of POD Mode Magnitudes (POM) when using POD, or Hankel Singular Values (HSV) when using BR.

Values (HSV). Using the reduction measure, a suitable time invariant projection operator P_k may be computed to generate the reduced order state variable using the projection relationship of,

$$\mathbf{z}(t) = P_k \mathbf{x}(t), \tag{3.5}$$

where $P_k \in \mathbb{R}^{k \times N}$. For the case studies presented, the method of Galerkin projection [Skogestad and Postlethwaite, 1996] is utilised for obtaining a reduced order representation of the form of Eq. (3.2).

Note on the Scope

Since the framework defined here is based on control-theoretic formalism, it can be broadly applied to any kind of system that can be modelled by Eq. (3.1). The two main stated objectives of this thesis are new algorithms for robotics and synthetic methods for validating motor control hypothesis in neuroscience.

As described in Sec. 1.4, the experiments focus on the linear systems of the tethered mass and the pendulum robot model and various flavours of nonlinear 2 link kinematic chains. In terms of behaviours, point-to-point reaching as well as simple repetitive periodic motions are studied. However the framework itself can easily be used to address more complex systems in performing more challenging tasks; this is however outside the scope of the investigations. The main studies carried out in this thesis are summarised next within the framework of reduced dimensionality analysis.

3.2 Natural Dynamics : $f(\mathbf{x}, t)$

The first study presented in Chap. 4 quantifies the relationship between the reduced dimensionality $\mathcal{D}$ and the natural dynamics of a system denoted by $f(\mathbf{x}, t)$. In the context of this thesis, the exploitation of natural dynamics of the system is demonstrated using two case studies. The term *'Natural Dynamics'* refers to the behaviour of the system when the inputs are of 0 magnitude [Carbajal, 2012]. From the state space description of the system of Eq. (3.1), natural dynamics thus refers to $f(\mathbf{x}, t)$.

In the first demonstration in Sec. 4.1 the relationship between reduced dimensionality and the passive mechanical properties of a system is studied. A mass-spring-damper system, is used as a model system as a loose analogue of vertebrate limb and the reduced dimensionality is quantified by using the POD algorithm. An analysis is then performed on the variation of the reduced dimensionality due to variation of stiffness, damping and mass along the limb in different configurations. The variation in physical parameters is taken as an idealised example of the growth process accompanying development.

The result of this exploration indicates that certain configurations of parameteric changes facilitate *'desirable'* reduced dimensionality changes. The increase of mass and stiffness in the proximal regions of the chain enables a progressive increase in the reduced dimensionality; this is similar to the natural phenomenon of proximo-distal growth in infants. The increase in reduced dimensionality driven by this parameteric change is similar to predictions for progressive DoF increase mechanisms accompanying development; The Bernstein sugggestion in this context is a mechanism of *unfreezing* of the DoF (see Sec. 2.2.1). Our results however indicate that the unfreezing witnessed in infant motor development might simply be a consequence of a change in the passive mechanical properties dictating the reduced dimensionality. It is however clear that reduced dimensionality is tightly coupled with the mechanical parameters specifying a system and thus appropriate mechanical design principles can be used to regulate reduced dimensionality. The theoretical underpinnings of the dimensional change problem as suggested by this exploration are further examined in the fifth study presented in chap. 8.

In the second demonstration, a control strategy is presented for the exploitation of this relationship between natural dynamics and reduced dimensionality. Inspired by the theories of motor primitives (see Sec 2.2.6), a developmental synthesis methodology for a reduced dimensional controller is proposed for compliant redundant robot platforms. In the proposed architecture, motor primitives are described as a constraint on the control input of the form,

$$\mathbf{u}(t) = W^* \tilde{\mathbf{u}}(t), \tag{3.6}$$

where W^* are the primitives in the form of a set of k muscle activations across all of the muscles (inputs). The new inputs to the system $\tilde{\mathbf{u}}(t)$ thus describe the combinators of the primitives. The motor primitives synthesised by the proposed method are fundamentally related to the natural dynamics since they are derived from a reduced order dynamical model of the system derived from POD; the perturbations employed mimic the natural process of spontaneous motor activity occurring during sleep. The relationship to the natural dynamic behaviour can be better understood by the proper orthogonal mode magnitudes, since they are directly related to the natural modes of a linear system [Kerschen and Golinval, 2002].

The reduced dimensionality of the system $\mathcal{D}$ in this case is thus derived from POD and directly specifies the number of primitives, i.e. k. The results of testing this controller are demonstrated on a simulated linear model of the pendulum robot. The controller computed using the primitives is designed to affect the equilibrium position of the end point. The results show how knowledge of the equilibrium position corresponding to the primitives can be utilised to generalise control to the whole workspace.

For both of these demonstrations, the natural dynamics thus directly dictates reduced dimensionality; the variation was studied in the first case while the control synthesis exploiting this phenomenon was presented in the second. However, since POD based methods are used, this reduction is described as an *intrinsic* reduction which is task-space independent. The next section describes the study exploring the role of the task-space.

3.3 Learnability and the Task Space : $y(\mathbf{x}, t)$

In this study, presented in Chap. 5, the relationship of reduced dimensionality $\mathcal{D}$ to the task space $\mathbf{y}(t)$ is studied in the biological context of behaviour development. As described earlier, reduced dimensionality is intrinsically related to the developmental problem of motor skill acquisition. Reduced dimensionality entails that simpler representations of a system exist. Therefore development could be viewed as the usage of increasingly complex representations in a task-specific manner. The importance of this mechanism can be seen in the difficulties in learning in high dimensional spaces. If optimal principles underlie motor control, dimensionality represents a constraint on the rate of learning. Reduced dimensionality allows a way of progressively regulating *learnability*. While initial explorations may be fast in low-dimensional spaces, the progressive increase allows the optimality of the computed solutions to increase as well.

In the first part of this study, a theoretical equivalence is established between two contrasting hypotheses of motor control theories : motor primitives (as defined in Eq. (3.6)) and norm minimising optimal control. While the former is a model for how the task control is simplified in a here-and-now perspective, the latter presents a mechanism for redundancy resolution. These two problems are however related to dimensionality and using the case of a simple linear system the equivalence of motor commands in both cases is demonstrated.

In the second part of this study, simulations are presented using a model of a vertebrate limb with elastic muscle-like actuation. The task space in this case is defined as the Cartesian position of the arm $[x_p, y_p(t)]^T$.The relationship of reduced dimensionality to the task performance is empirically quantified for 2 kinds, behaviour that reach positions in space, and behaviours involving simple repetitive patterns at the output. Using a nonlinear reduction algorithm of empirical balancing, the reduced dimensionality is obtained unique to the specification of the Cartesian endpoint positions denoting the task space. The variation of reduced dimensionality thus results in progressively increasing performance in tasks. This is however task dependent due to the nature of the reduction analysis. Thus progressive change in learnability can be utilised for improving task performance in a task-dependent manner.

3.4 Muscle Synergies : $g(\mathbf{x}, \mathbf{u}, t)$

In this study, presented in Chap. 6, the relationship of the reduced dimensionality $\mathcal{D}$ to the input $g(\mathbf{u}, t)$ of a system is examined under the perspective of the muscle synergy hypothesis. Muscle synergies are defined as coordinated activations of muscles. It has been suggested that the mechanism employed by the CNS in simplifying the control problem is to modularise the control input by grouping together muscle activations; spatio-temporal regularities observed in EMG and kinematic data of human movements are cited as evidence (see Sec. 2.2.6). In particular, one such model is the temporal synergy formulation, in which the control input is costrained to the form,

$$\mathbf{u}(t) = \hat{W}\Psi(t), \tag{3.7}$$

where $\Psi(t)$ is the synergy comprising of a task-independent set of basis patterns, $\hat{W}$ the weight matrix is a the task-dependent linear combination of synergies. The muscle synergy hypothesis contends that the motor coordination problem is directly simplified through input dimensionality reduction. Although the synergy hypothesis explains a number of observed features in the inter-muscle coordination in movement, it has faced criticism for being a phenomenological model of observed behaviour. A bottom-up approach to examining the reduced dimensionality in behaviour due to synergistic control can provide testable conditions for the validation of this hypothesis. In this study two methods are developed towards quantifying this relationship : Trajectory Specific Dimensionality Analysis (TSDA) and Minimum Dimensional Control (MDC).

The proposed TSDA framework analyses the the relationship between a particular synergy basis and the reduced dimensionality of the system. The formulation of Eq. (3.7) represents a modification of the control input $g(\mathbf{x}, \mathbf{u}, t)$. The insight underlying this study is that under synergistic control, the behaviour of the system can be described by the dynamics of an equivalent system comprising of the weights and actuated by the synergy basis patterns. This is a constrained-reformulation of the system dynamics; by quantifying the reduced dimensionality, the comparison of the dimensionality of behaviour is possible. This quantification is thus a task, and synergy specific measure of the reduced dimensionality of a system. A measure using HSV is developed for quantifying the reduction in dimensionality in following a particular trajectory. The experiments on both linear and nonlinear systems compare various trajectories in terms of reduced dimensionality, i.e complexity of learning and generating the control.

The second outcome of this study is a Minimum Dimensional Control (MDC) model which incorporates the TSDA in an optimisation scheme. This method computes the optimal reduced-dimensional weight matrix (i.e. the optimal trajectory) for a given system employing a given set of synergies, while meeting the task constraints. Using a HSV measure of reduced dimensionality a cost function is derived for minimum dimensionality. The experimental results show that the numerical optimisation of this cost for a reaching task resulted in sigmoidal straight-line trajectories with symmetric bell-shaped velocity profiles as the minimum dimensional solution. The results were replicated in both a linear and a nonlinear model system and were tested on synergies defined by Legendre polynomial and Fourier bases. This result constitutes an important outcome of both this study and this thesis, since it is indicative of reduced dimensionality principles underlying motor control.

3.5 Dynamical Movement Primitives (DMP) : $g(\mathbf{x}, \mathbf{u}, t)$

In this study, presented in Chap. 7, the TSDA analysis method proposed in Chap. 6 is extended to quantify the relationship between reduced dimensionality $\mathcal{D}$ and control using the DMP. DMP is a control architecture that utilises a mechanism of tunable nonlinear dynamical systems (see Sec. 2.3.5).This control strategy has increasingly found use in the control of high dimensional robots for applications like imitation learning from human motions. Quantifying the reduced dimensionality is therefore useful both from a perspective of human movement analysis as well as expediting the existing online learning approaches for training the DMP. However, the analysis of reduced dimensionality utilising the TSDA on this architecture is not straightforward since the basis set is now of the form,

$$\mathbf{u}(t) = f_{DMP}(\hat{W}, \Psi(t, \mathbf{x})), \tag{3.8}$$

where the functions $\Psi(t, \mathbf{x})$ are the outputs of the nonlinear dynamical system underlying the DMP system. The primitives in this case are state dependent and it is not obvious how the linear decomposition can be accomplished rendering the task-independent basis and the task-dependent weights.

In this study, this problem is treated by analytically solving the DMP equations and decomposing the signals to obtain a set of basis patterns in the form of Eq. (3.7); a method of Iterative Basis Extraction (IBE) was developed for obtaining the basis patterns computationally. The result of this process is that trajectory comparison can be performed when using DMP control.

The comparison of the task-speccific reduced dimensionality from DMP control is demonstrated on a linear spring-mass-chain systems of increasing number of elements, and compliant model of a quadrupedal robot leg. Three kinds of tasks are compared in the first set of experiments. The results indicate that when using DMP control, some trajectories result in greater reduced dimensionality. Furthermore, the degree of decrease increases with the dimensionality of the original system - spring mass chains with large number of elements performing smooth (sigmoidal) end-point reaching tasks show a large degree of reduced dimensionality. Moreover, the corresponding reduced dimensional models synthesised for each of these tasks corresponds closely with the full dimensional systems. Thus, the proposed method enables the synthesis of task-dependent reduced dimensional models that capture the input-output behaviour to the best degree.

The application of the proposed method to robotics is demonstrated using a simulated model of a compliant quadruped leg inspired by the Cheetah robot of the EPFL. The quantification of the reduced dimensionality for various foot stepping tasks is presented in the results. This study is thus a demonstration of the strategy of exploitation of reduced dimensionality for biomimetic robotics applications.

3.6 Development and Dimension Change : $\delta\mathcal{D}(\mathcal{F})$

In the final study, presented in Chap. 8, Bernstein's [Bernstein, 1967] notion of DoF change accompanying the development of motor skills in organisms is examined. DoF change is formalised as a problem of reduced dimensionality change. The problem entails a study of factors that result in a ordered reduced dimensionality change sequence $\mathcal{D}_i$. Parametric variations in the natural dynamics $f(\mathbf{x}, t)$, input function $g(\mathbf{x}, u, t)$ and task requirement $y_d(\mathbf{x}, t)$ are examined in detail.

In this formalisation, the impact on progressive task complexity increase due to natural dynamics changes are first studied. Simulation results demonstrate that some forms of passive mechanical property changes can directly impact task complexity increase due to increase in the reduced dimensionality. The '*growth*' of passive mechanical properties is presented as a trajectory in a parametric space resulting in motor skill changes. The improvement of skills thus implies optimal growth strategies within this parametric space. These results thus demonstrate how the growth process itself can regulate learnability, i.e. the reduced dimensionality $\mathcal{D}$.

This approach connects the results presented in the studies in chap. 4 and in chap. 5 within a common framework. This study holds important implications for the developmental notion of progressive increase in an organisms skills and abilities – the Bernstein notion of DoF increase (see Sec. 2.2.1). From the perspective of developmental robotics, the results demonstrate how morphological changes, such as through variable compliance, can be used to directly regulate the rate of acquisition of various motor skills.

Chapter 4

Reduced Dimensionality and Natural Dynamics

In this chapter, the relationship between natural dynamics and reduced dimensionality is examined in two demonstrative examples. The primary research question under investigation is:

1. how are the natural dynamics of a system related to its reduced dimensionality? How can this relation be exploited?

4.1 Passive Properties and Dimensionality

This section summarises the research presented in [Kuppuswamy et al., 2012a] which can be found in Appendix A. In the context of this thesis, the paper investigated the following research question:

1.a. What is the effect of the passive mechanical properties on the reduced dimensionality of a simple linear system?

Kuppuswamy, N. , Harris, C. M., Cangelosi, A. (2013). **Effect of physical variation on the reduced dimensional control of a mass-spring-damper chain system**, in Proc. of the IEEE Conference on Development and Learning and Epigenetic Robotics, San Diego, USA, 2012.

Abstract : *In this work the relationship between growth and ontogenetic development of motor control is studied from the perspective of reduced dimensionality in control; such motor control strategies have been suggested as a possible mechanism circumventing the degree of freedom coordination problem. The relationship between reduced dimensional behaviour and parametric variation is empirically analysed in a simulated actuated mass-spring-damper system, as a loose analogy to physical growth process in vertebrate limbs. The resultant dimensionality change is analysed and ideal directions for growth, in terms of physical parameter variations, are discussed.*

One of Bernstein's notions on development correspond to the dimensional change during growth and skill learning (see Sec. 2.2.1). In a robotic context, this has been applied in models using a gradual and incremental increase in the DoF for sensorimotor exploration in solving tasks [Berthouze and Lungarella, 2004]. While it is known that the natural growth process is accompanied by a variation in passive mechanical properties, such as changes in mass, stiffness and damping, it is not really known how ontogenetic processes relate to such changes.

This study presents two main observations in addressing this question. First, the dimensionality in the kinematic sense Bernstein is instead quantified as the reduced dimensionality of the embodiment. Second, the concept of dimension change is then examined as the effect of parameteric variation in the passive properties on the reduced dimensionality. Furthermore, this study explores different configurations of parameteric changes in order to mimick natural proximodistal growth processes. This demonstration relies on theoretical results relating POD to the natural dynamics. POD is a well understood statistical model reduction technique as reviewed in Sec. 2.4.2. It has been shown that the proper orthogonal modes of a linear system are directly related to its normal modes [Kerschen and Golinval, 2002]. Based on this equivalence, an adaptation of the passive mechanical properties which leads to a adaptation of the normal modes. This can be intrepreted as a variation in the reduced dimensionality itself through a threshold on the POD measure.

Methods and Results

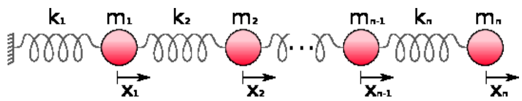


Figure 4.1: Chain of mass-spring-damper elements tethered to one end and free to move in the other. The movement is constrained to a $1D$ line.

The experiments were performed on a simulated chain of mass-spring-damper depicted in Fig. 4.1. The chain was free to move in one end (distal end). The chain was perturbed with step inputs and the proper orthogonal modes of the state dataset were analysed. Reduced dimensionality was then derived by using a threshold measure as seen in Fig. 4.2b. This process is then repeated for various configurations of physical parameters. The variations in parameters results in a change in the state dataset obtained from spontaneous motor activity as depicted by the colour changes in Fig. 4.2a. The changes in the POD magnitudes is seen in Fig. 4.2b, resulting in the variation of the reduced dimensionality seen in Fig. 4.2c. The dynamic representations that correspond to the reduced dimensional models in each case can be used for control as well. A model based control for following a smooth sigmoidal trajectory that uses these reduced dimensional models is seen in Fig. 4.2d

The results show that damping decrease has the most direct contribution to an increase in reduced dimensionality. Since the POD study was performed on normalised thresholded proper orthogonal mode magnitudes, it can be seen that damping decrease that occurs proximally (close to the base) and then progressively outwards results in a uniform reduced dimensionality increase. This observation when coupled with the results of mass increase that is proximodistal in structure corresponds to an increase in reduced dimensionality that is similar to the Bernstein model of DoF increase.

Although the results are demonstrated on a simplified linear system, it can be seen that reduced dimensionality is a good model for studying development of motor skills. In artificial

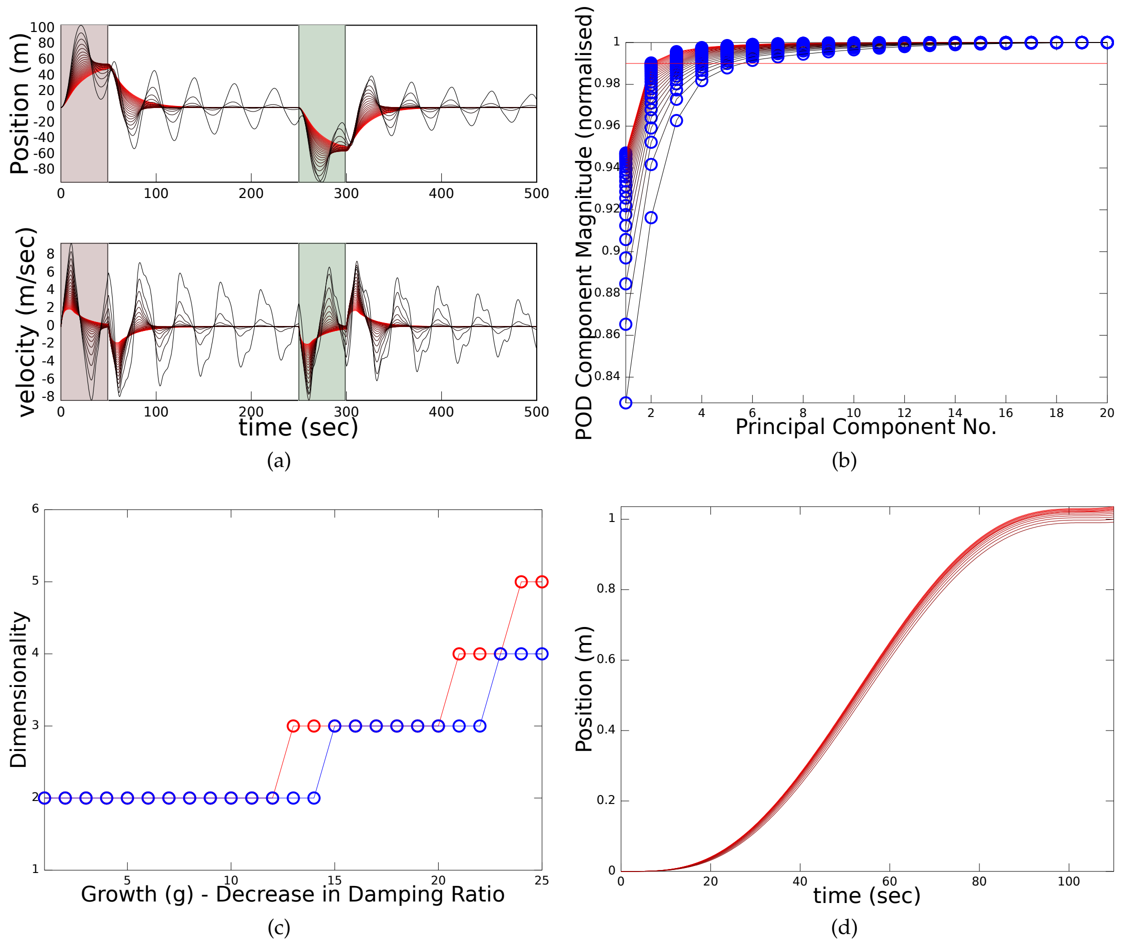


Figure 4.2: Change in reduced dimensionality in a mass-spring-damper chain with parameter variation : (a) variation in state trajectory for step perturbations - decreasing damping proximo-distally indicated with darkening colours; (b) shift in the magnitudes of normalised proper orthogonal modes for variations in damping - the red flat line is the threshold for quantifying reduced dimensionality; (c) gradual increase in reduced dimensionality; (d) Similarity of trajectories when using a controller incorporating the reduced dimensional model - sigmoidal reaching task performance.

systems, such approaches can be used in conjunction with variable compliance devices which can gradually adapt physical parameters alongwith the development of skill. This however is a preliminary result and Chapter 5 discuss the relationship to increasing task complexity, and the Chapter 8 presents a consolidated theory for this notion along with results on a nonlinear vertebrate limb model.

4.2 Motor Primitives for a tendon driven platform

This section summarises the research presented in [Kuppuswamy et al., 2012b] which can be found in Appendix B. In the context of this thesis, the paper investigated the following research question:

1.b. How can the reduced dimensionality be exploited for simplifying control in redundant compliant robots?

Kuppuswamy, N. , Marques, H. G., Hauser, H. (2013). **Synthesising a Motor-Primitive Inspired Control Architecture for Redundant Compliant Robots** , in From Animals to Animats - Proc. of the International Conference on Simulation of Adaptive Behaviour, Odense, Denmark, 2012.

Abstract : *This paper presents a control architecture for redundant and compliant robots inspired by the theory of biological motor primitives which are theorised to be the mechanism employed by the central nervous system in tackling the problem of redundancy in motor control. In our framework, inspired by self-organisational principles, the simulated robot is first perturbed by a form of spontaneous motor activity and the resulting state trajectory is utilised to reduce the control dimensionality using proper orthogonal decomposition. Motor primitives are then computed using a method based on singular value decomposition. Controllers for generating reduced dimensional commands to reach desired equilibrium positions in Cartesian space are then presented. The proposed architecture is successfully tested on a simulation of a compliant redundant robotic pendulum platform that uses antagonistically arranged series-elastic actuation.*

Biomimesis in morphologies has lead to a large interest in recent times for robots comprising of tendon driven and compliant mechanisms. However the large redundancy in such mechanisms necessities novel approached to achieving control perhaps inspired by neural mechanisms. There is however substantial evidence for modular organisation of the motor control in vertebrates (see Sec. 2.2.6) and translating some of these ideas into a robotic context could prove beneficial.

This study extends a recently proposed model for biological motor primitives which mimic Convergent Force Fields (CFFs) in frogs. The underlying principle is that motor primitives can be derived from a reduced dimensional representation of a system [Berniker et al., 2009]. This study is aimed at deriving a computationally efficient and developmentally acquired control strategy based on the definition of motor primitives.

Methods and Results

The biological synthesis model proposed in [Berniker et al., 2009] suggested that motor primitives for an organism are derived from a reduced dimensional representation of its dynamics by utlising two kinds of criteria : (a) the primitives must span the space of inputs, (i.e. act as a reduced dimensional basis set) and (b) they project to the best extent possible into the input space of the reduced dimensional system (i.e. must be useful for computing control solutions). Since

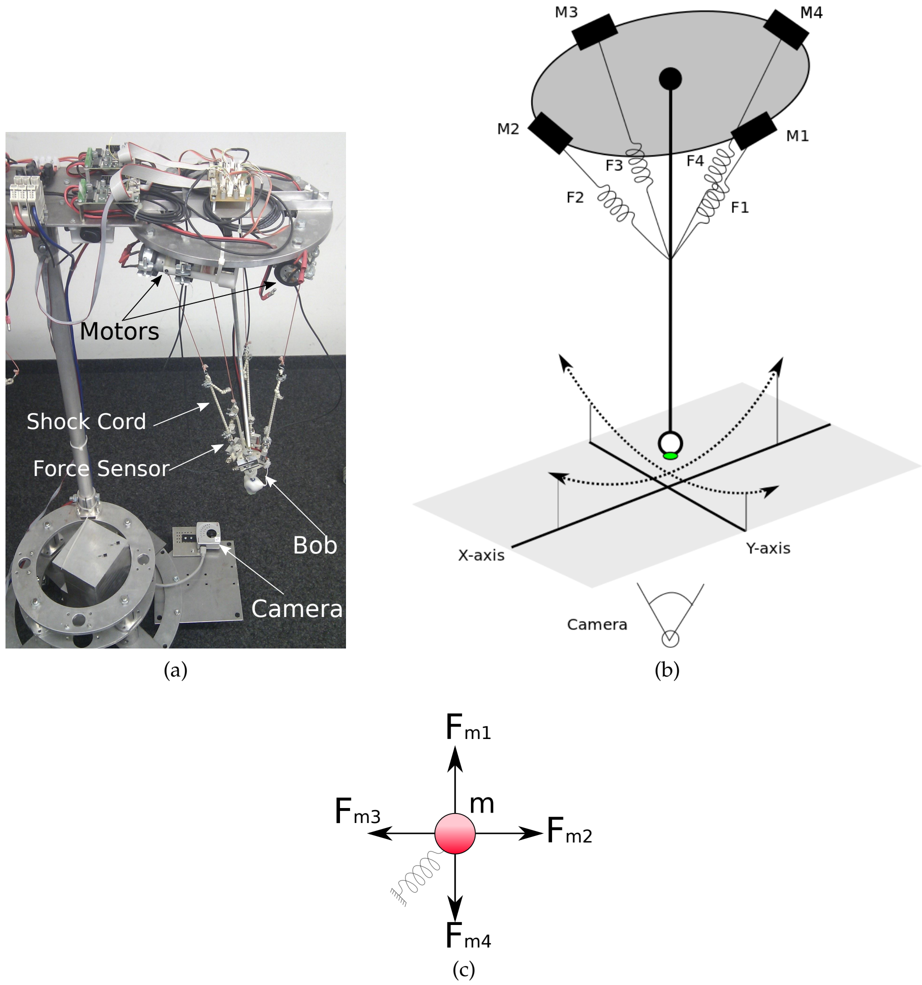


Figure 4.3: a) Pendulum robot platform; b) schematic of setup (reproduced from [Marques et al., 2012]); c) linear system simulation of the pendulum robot platform (the mass is that of the end-point bob)

motor commands in biology are typically specified by the activation of muscles, which is by definition non-negative, the synthesis in the biological model utilised a constrained optimisation for computing the basis set which best spans the space of inputs.

In the study presented in this section, a simplified synergy computation method that results in a similar criteria is derived using direct SVD based computation - the proposed synthesis exploits the fact that actuators in robots can accept negative inputs too (typically for DC motors this results in a change in rotational direction). Using this definition a computationally efficient motor primitive synthesis technique is derived based on SVD calculations.

The experiments were performed on a simulated version of the pendulum robot platform - a tendon driven compliant pendulum system shown in Fig. 4.3. The robot was modelled as a linear system consisting of a mass anchored to the origin by spring and damper element and constrained to move in a $2D$ space. The linear dynamical actuation model was also utilised to mimic the behaviour of the series-elastic muscle-like actuators. The linear approximation of the system is based on the observed behaviour of the end point of the pendulum; a camera pointing upwards gathers the data. The approximation holds for bounded motion of the endpoint (bounded within $30cm$ diameter from the rest position).

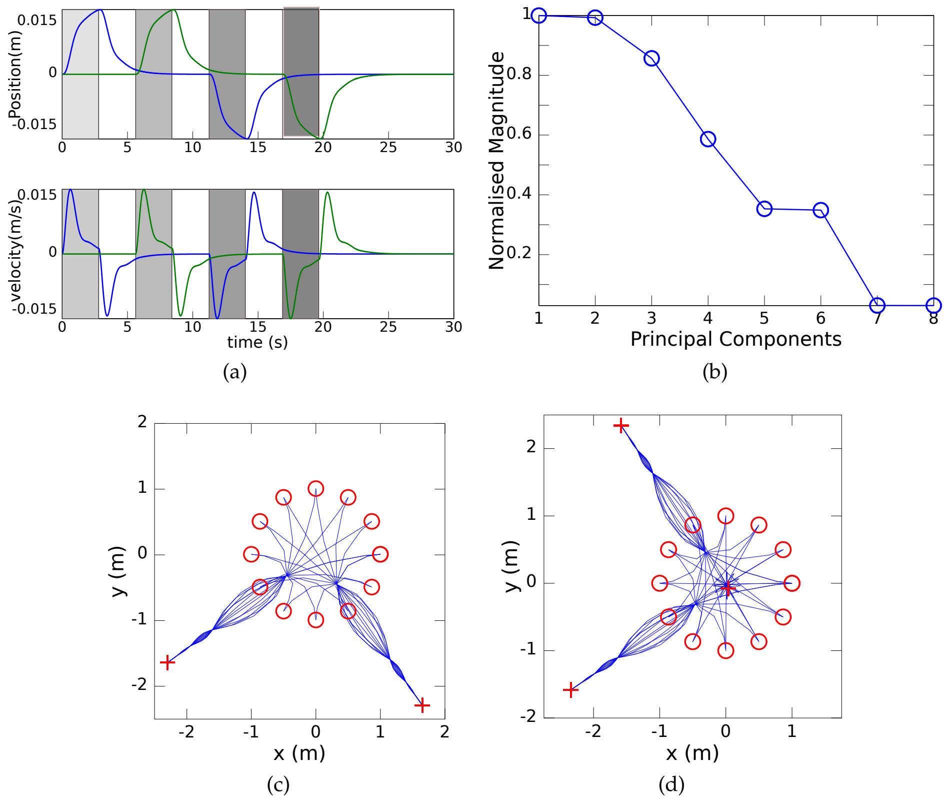


Figure 4.4: a) Spontaneous motor activity and trajectory : reduction to a k dimensional system allows computation of k unique equilibrium points from which control can be computed by interpolation; b) normalised proper orthogonal mode magnitudes; equilibrium point solutions for (c) $k = 2$, and (d) $k = 3$. The red circles are initial conditions, the blue lines are trajectories to the equilibrium positions which are represented by red crosses.

The algorithm that was proposed also takes inspiration from the biological process of Spontaneous Motor Activity (SMA); the reduced order model is obtained using POD from a state trajectory resulting from SMA like perturbation. This process can be seen in Fig. 4.4a, b. The primitives produced by this technique are visualised in Cartesian space (i.e. the output space) in the form of the equilibrium positions, the results for 2 and 3 primitives is shown in Fig. 4.4c, d. The knowledge of these equilibrium positions allow the computation of weights for linearly combining the primitives to reach novel tasks in the workspace.

4.3 Summary and Conclusions

This chapter presented two studies which explore the relationship of natural dynamics of a system to its reduced dimensionality. The first study investigated how passive mechanical properties directly are related to reduced dimensionality; this is quantified by using the POD method. The variation of physical parameters mimicking the growth process accompanying development results in a variation of this reduced dimensionality. This result is then used to derive conclusions on the principles underlying dimensional change during development of motor skills. The Bernstein notion of DoF increase could thus be an artefact of the increase in reduced dimensionality due to the growth and strengthening of the musculo-skeletal apparatus. This result has implications for artificial systems as well. Since reduced dimensionality can be regulated by passive properties, a similar approach could be used for progressively increasing the complexity of a robot. Recent developments such as variable compliance actuators could be exploited to enable this increase. The theoretical underpinnings of the results presented here are discussed in greater detail in chap. 8.

The second study presents a method for exploiting the relationship between natural dynamics and reduced dimensionality through the synthesis of simplified control strategies that are inspired by biological motor primitives. The natural dynamics in this case dictate the number of primitives that are needed to capture the space of inputs with a reduced dimensional control. The proposed synthesis framework also takes inspiration from the spontaneous motor activity. A computationally efficient developmental model is proposed for the synthesis of primitives that exploits the nature of artificial actuators used in robotic systems. The proposed method was tested on a linear simplified representation of a tendon driven compliant robot platform - the pendulum robot. From the primitives, equilibrium positions are then computed to convert the control problem into the task-space, novel control solutions result from linear combination of the primitives corresponding to these positions.

It must be noted that the usage of POD in both these methods allows the quantification of a task-independent reduced dimensionality, or an *intrinsic* reduced dimensionality. Although a task-specific reduced dimensionality is ideally required, the methods in this formulation can be exploited through the regulation of passive mechanical property variations; as mentioned earlier, variable compliance systems seem ideal for this purpose. Lastly, only relatively simple linear systems are studied using these methods. The subsequent chapters in this thesis study the nonlinear variation of the reduced dimensionality problem. Nevertheless, the results presented here form a useful source of inspiration for the explorations presented in the subsequent chapters.

Chapter 5

Reduced Dimensionality, Learnability and the Task-Space

This chapter summarises the research presented in [Kuppuswamy and Harris, 2013] which can be found in Appendix D. In the context of this thesis, the paper investigated the following two research questions.

2.a. How is reduced dimensionality related to hypothesis for motor control, in particular to optimisation approaches?
2.b. How can task-specific reduced dimensionality be exploited in a system?

Kuppuswamy, N. , and Harris, C. M. (2013). **Developing Learnability - the Case for Reduced Dimensionality**, in Proc. of the IEEE Conference on Development and Learning and Epigenetic Robotics, Osaka, Japan, 2013.

Abstract : *In this work, the notion of reduced dimensionality and its relevance for systems undergoing development is examined. The various motor control theories of degree of freedom change, optimal control, and motor primitives are related using the framework of control dimensionality reduction. Based on their relationship, we propose a developmental approach based on progressively utilising increasingly higher dimension representations of the system. A simulated planar 2 link arm model is then used to demonstrate the effect of utilising reduced dimensional models for control; comparisons on step and sinusoidal tasks are presented showing a progressive decrease in error that is task dependent quantitatively. Arguments are presented for why such a strategy might be essential from an evolutionary perspective for the developmental acquisition of motor control in a tractable manner.*

As reviewed in Sec. 2.2, the DoF problem lies in understanding how redundancies are resolved in the neural control of movements. The redundancy in a set of solutions for solving a motor task is however also related to the high-dimensional state-space in which behaviour resides. The hypotheses for the principles underlying motor control can be grouped under three broad categories : (1) modular control principles, (2) optimisation of the motor control and (3) developmental motor skill acquisition. Although all three suggest some form of dimensionality reduction underlying the motor control mechanisms, the relationship between these hypotheses is not obvious.

In the first part of this study, a formulation of the problem on a linear system is used to show how motor primitives and norm minimising control both equivalent in terms of the input computed for a task. The approach uses a reduced dimensional model of a system through a projection where the reduced dimensionality is same as the task space dimensionality. From this reduced dimensional system, motor primitives can be extracted using the approach discussed in chap. 4. Since the number of primitives is then given by the task-space dimensionality and the reduced dimensional model minimises a projection norm, an identical control input is computed by this approach on comparison with the minimum norm solution.

The second exploration in this study is based on the notion of learnability, which is a constraint on the rate of learning introduced by high dimensionality. Thus from a behavioural perspective dimensionality reduction allows increased rates of learning, however the tradeoff is in the sub-optimality of solutions that could be acquired. However, if a task-specific dimensionality reduction approach is employed, a progressive change in dimensionality can lead to a task-specific improvement in performance. Thus reduced dimensionality needs to underlie the development of motor skills.

5.1 Methods and Results

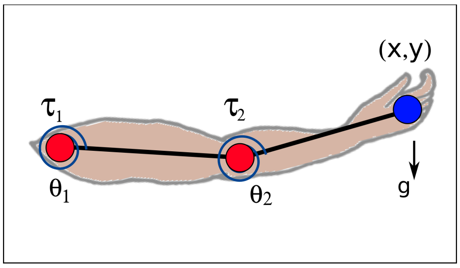


Figure 5.1: Model of arm used for experiments : 2-link kinematic chain with joint compliance and damping, actuated by muscle-like actuators with first-order response.

For the experiments, a compliant vertebrate limb modelled as a 2 link kinematic chain system of Fig.5.1 is utilised. Muscle-like actuators with first order dynamics are used to apply joint torques. The system has state dimensionality of $N = 10$ and the model incorporates gravity. Nonlinear balancing about the rest position (under no activation) is used to compute a set of equivalent reduced dimensional models of increasing dimensionality (i.e. reduced dimensionality of $3 \leq k \leq 10$).

In order to test the viability of the reduced dimensional models for control, the synthesised reduced dimensional models are subjected to step and sinusoidal perturbations and the resulting cartesian trajectories are seen in Fig.5.2a. The The result presented in Fig.5.2b is a measurement of Cartesian end position difference (error) between the reduced and full dimensional models; error is measured throughout the whole trajectory. The trend of progressive task-specific error decrease is accompanies the dimensionality increase. This result implies that if reduced dimensional models were of progressively increasing dimensions were utilised (implicitly or explicitly) for control and optimisation of movements, at different stages of development performance improvements will be seen. The task-specificity implies that at different stage of development the learnability can be regulated specifically to task requirements.

An important consequence of the need to optimise the motor skill development is that there is a progressively increasing number of degrees of freedom employed on more complex tasks although this increases the time for skill acquisition; this is also observed in human skill development. The results demonstrated in this study indicate that increasing task complexity may be related to employment of increasing dimensions in the control, without compromising on the stability of control of tasks that are already acquired.

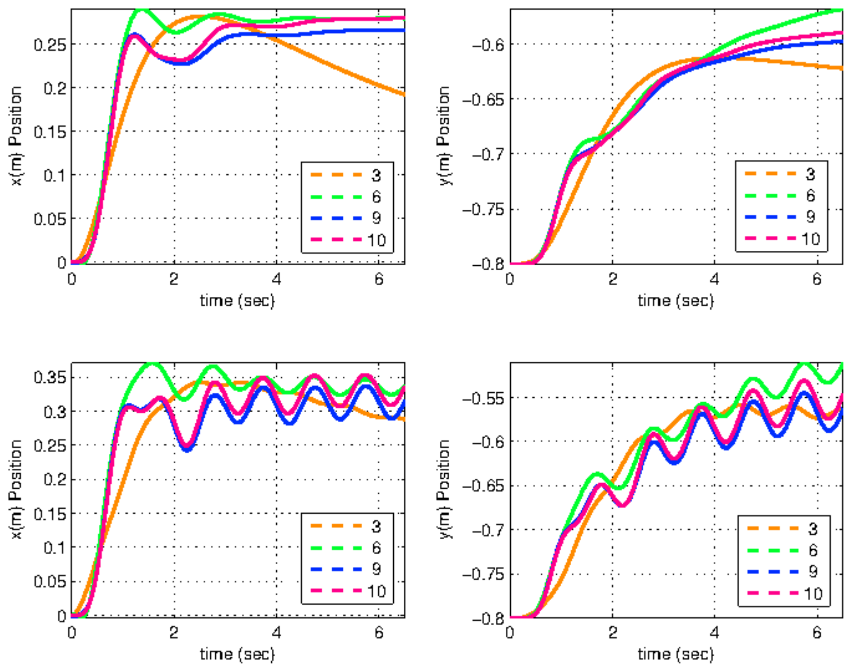
5.2 Summary and Conclusions

In this study, the relationship of reduced dimensionality to task specification and requirements was studied. The first part of this study showed how the various flavours of motor control hypothesis might be equivalent; explicit or implicit dimensionality reduction in the neural mechanisms is suggested as a consequence of many of these hypotheses. Extending this notion to behaviour performance requires the understanding of task-specific reduced dimensionality; the broad need for such a principle was also discussed in chap. 1.

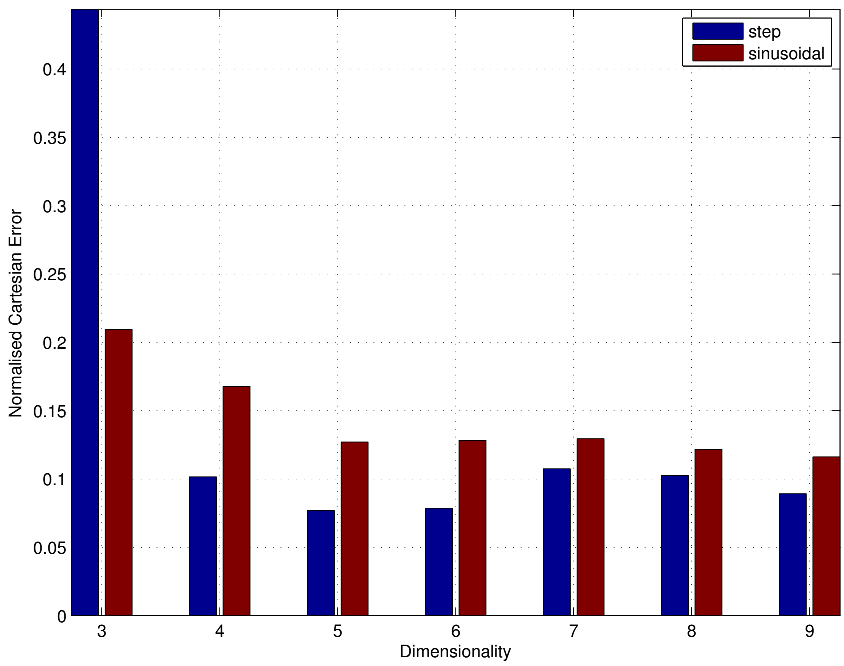
The importance of task-specificity is evident from the optimisation theory viewpoint on motor control. If optimal behaviour performance is sought, for even moderately large state space dimensionality the search may be intractable. Thus from a developmental perspective, the rate of learning, or learnability, is constrained by the dimensionality. Reduced dimensionality presents a mechanism to circumvent this problem since the learning rate can be speeded up, although this trades of against suboptimality in the solutions.

However, task specificity in the reduced dimensionality can allow individual tasks to be acquired at different learning rates depending on the respective performance levels that are necessary at any given stage of development. For example, in the case of human movements, there is a trend towards improvements in reaching behaviour performance prior to other advanced manipulation abilities. However, in adults there is clearly a need to adapt and acquire specific advanced manipulation skills in a more real-time perspective.

The approach and results presented in this study indicate how reduced dimensionality can enable this mechanism through the relationship to the task-space. In a robotics perspective, the method of empirical balancing has potential for use in simplifying the control. The approach presented in this chapter point to a strategy for developmental acquisition of skills in high dimensional robots, such as anthropomimetic systems by incorporating the task-space perspective of reduced dimensionality.



(a)



(b)

Figure 5.2: Comparison of trajectories of full and reduced order models; (a) step response, (b) sinusoidal response; full and reduced order trajectories (dimensionality $3, 6$, and 9) are dashed lines of varying colours; (c) error in step and sinusoidal responses of the reduced dimensional system decreases with increase in dimensionality.

Chapter 6

Muscle Synergies and Reduced Dimensionality

This chapter is based on the work reported in [Kuppuswamy et al., 2013] which can be found in Appendix D. In the context of this thesis, the paper investigates the research question :

3. How is the input actuation of a system related to its reduced dimensionality?

Kuppuswamy, N. , and Harris, C. M. (2013). **Do muscle synergies reduce the dimensionality of behaviour?** , Frontiers in Computational Neuroscience (under review).

Abstract : *The muscle synergy hypothesis is an archetype of the notion of Dimensionality Reduction (DR) occurring in the central nervous system due to modular organisation. In order to validate this hypothesis, it is important to understand if muscle synergies can indeed facilitate accurate real-time control and optimisation of motor behaviour. In this paper, we investigate this problem by synthetically examining the reduction of spatio-temporal behaviour dimensionality due to control using muscle synergies. Our approach is based on the observation that control in the form of temporal muscle synergies constrain the dynamic behaviour of a system in trajectory-specific manner due to the synergy weight matrix. We then use system balancing to define a normalised Hankel Singular Value (HSV) measure for quantifying the DR of this constrained system; we term this approach as Trajectory Specific Dimensionality Analsyis (TSDA). We then develop a model for Minimum Dimensional Control (MDC) to find the optimal weight matrix corresponding to the minimum dimensional trajectory that satisfies all of the task constraints. The TSDA and MDC methods are tested on simulation on linear (tethered mass) and nonlinear (compliant kinematic chain) system; dimensionality of various reaching trajectories is compared and idealised synergies of Legendre polynomials, and Fourier bases are tested. We show that smooth straight-line Cartesian trajectories with bell-shaped velocity profiles emerge as a minimum dimensional solution to reaching tasks in linear and nonlinear systems. The results indicate that a system, synergy profile and trajectory-specific DR of motor behaviours results from usage of muscle synergy control. The implications of these results for the synergy hypothesis, optimal motor control, and developmental skill acquisition are then discussed.*

In this study, the following problems are examined : (i) how can the behaviour of a dynamical system under modular control in the form of synergies be quantified in terms of dimensionality and how to quantify the task-dependence, if any? (ii) Can this task-dependent dimensionality be minimised by an appropriate combinations of synergies and if so what is the resultant behaviour? Using the framework of reduced dimensionality analysis two methods are developed towards answering these questions : (a) Trajectory Specific Dimensionality Analysis (TSDA), and (b) Minimum Dimensional Control (MDC).

Muscle synergies are defined as coordinated activation of muscles in order to accomplish tasks. Many formulations of such muscle synergies have been proposed through observations of spatio-temporal regularities in human motor behaviours (see Sec. 2.2.6). In this context, the temporal synergy formulation decomposes the control input into a linear task-dependent weighted combination of task-independent set of synergies acting as a basis set. This formulation has developmental underpinnings and is ideal for examining the temporal complexity of behaviour.

The TSDA approach is motivated by the observation that the behaviour (state trajectories) of a dynamical system driven by combinations of temporal synergy patterns for the duration of a behaviour can be described by an equivalent trajectory-specific constrained-reformulated system. This is a virtual dynamical system that exists for the duration of the movement and its dynamics are dependent on the magnitudes of the synergy weight matrix. For a given task, the weight matrix is computed using an inverse dynamic model and a least-squares approximation using the synergy basis. However, the trajectory that results is non-unique. The reduced dimensionality of this constrained-reformulated system is then quantified using balanced reduction; the method identifies a control relevant subspace on the state of the system that captures most of the input-output dynamics; input is this case given by synergy basis and output is given by the task-space specification. The task-dependent, trajectory-specific dimensionality is then quantified using a Hankel Singular Value (HSV) measure and an appropriate threshold.

In MDC, a control model is proposed for finding the minimum dimensional trajectory that satisfies task objectives. Using the TSDA, a smooth real-valued dimensionality cost function that is based on the HSV is proposed. Through a constrained minimisation of this cost function, it is shown that for a given system, and a given set of synergies, MDC yields the optimal dimensional synergy weight matrix for a specified task. The MDC control when applied to reaching tasks on linear and nonlinear systems obtains trajectories with smooth sigmoidal straight lines and bell-shaped velocity profiles similar to humans.

6.1 Trajectory-Specific Dimensionality Analysis (TSDA)

The generic dynamical system given by Eq. 3.1, utilising control in the form of temporal synergies can be represented by,

$$\mathbf{y}(t) = h(\mathbf{x}, t), \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + \hat{g}(\mathbf{x}, \Psi, t), \tag{6.1}$$

This is termed as a constrained-reformulation of the system dynamics where the inputs are the temporal synergies $\Psi(t)$, and can be viewed as signals which control the onset and termination of the movements for a task. For the duration of the behaviour, the dynamics is thus described by Eq. (6.1) due to the constrained input function $\hat{g}(\,\cdot\,)$ as,

$$\hat{g}(\mathbf{x}, \Psi, t) = g(\mathbf{x}, \hat{W}\Psi, t). \tag{6.2}$$

It must be emphasised that the constrained-reformulation only describes a '*virtual*' system dynamics for the duration of the movement when actuated by the synergistic input $\Psi(t)$. The state-space however has not changed; i.e. the state variable x for constrained-reformulated system is the same as the original system. The system of Eq. (6.1) is then denoted by $\hat{\mathcal{F}}(f(\,\cdot\,), \hat{g}(\,\cdot\,), h(\,\cdot\,))$.

Clearly, $\hat{F}$ is unique to a given trajectory and given synergy basis set, since it incorporates the weight matrix $\hat{W}$ corresponding to a trajectory T and uses input signals in the form of temporal synergies. Therefore, $\hat{F}$ is a trajectory specific constrained-reformulation of the dynamics. Then the trajectory specific dimensionality is given by,

$$\mathcal{D}(\hat{\mathcal{F}}) = D_T, \tag{6.3}$$

In this formulation, although any reduction can be utilised for computing $D_{\mathcal{T}}$, system balancing and the HSV based approach is used due to its relevance for the control problem. HSVs measure the importance of each of the state variables of the system $\hat{\mathcal{F}}$ or both the outputs (the task) and the inputs (synergy patterns). Thus they quantify the DR of the behaviours that is dependent on the kind of synergy patterns used. In order to make the comparison task magnitude independent, a cumulative normalisation of the HSVs is then used to quantify the trajectory-specific reduced dimensionality.

6.2 Minimum Dimensional Control (MDC)

The MD control proposal is to compute the weights which minimise the reduced dimensionality measure defined in Eq. 6.3 which also satisfies the task requirements. The optimisation problem of MDC can be stated as follows,

$$\begin{aligned} \hat{W}^*_{\mathcal{T}} = \underset{\hat{W_T}}{\operatorname{argmin}} \quad & J(\mathcal{D}_T), \\ \text{subject to} \quad & \dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, \mathbf{u}, t), \\ \mathbf{y}_{\mathcal{T}}(t_d) = \mathbf{y}_{\mathcal{T}t_d}, & \dot{\mathbf{x}}_{\mathcal{T}}(t_d) = \dot{\mathbf{x}}_{\mathcal{T}t_d} \end{aligned} \tag{6.4}$$

where $\hat{W}*_{\mathcal{T}}$ is the optimum weight combinator for a given system $\mathcal{F}$ and given task $\mathcal{T}$ that minimises the DR of the equivalent composite system. A computational solution is sought in the paper in Appendix E, therefore this cost function needs to be continous and efficient to compute from $\hat{F}$. Since the magnitude of the HSVs are continous, they are utilised for formulating this cost function.

6.3 Methods and Results

To demonstrate the proposed framework, two kinds of simulations were used : a linear system of a tethered mass and a nonlinear system of a kinematic chain as in Fig. 6.1. The results reported here are for the linear system (please refer to Appendix D for the nonlinear results). The linear system consists of a point mass moving in $2D$ space, tethered to an origin by a linear spring. The mass is subject to actuations in the form of independent forces in 2 orthogonal directions, apart from linear damping forces, and the output is the position in the $2D$ space relative to the origin.

The simulation was performed on Matlab 2012 using the *ODE* package. The equations were integrated using the *ode15s* solver with the settings, absolute tolerance = $5e^{-2}$ and relative tolerance $1e^{-3}$. The weights $\mathcal{W}$ for the benchmark tasks and the MDC problem were initialised by using the *fit* routine in the curve fitting toolbox for the polynomial and fourier bases. The numerical optimisation of the MDC was carried out using the *fmincon* routine, with the *interior point* algorithm.

The proposed methods are intended to be used on real synergy data; in this study, 2 kinds of idealised temporal synergies of orthonormal basis functions : (a) Legendre polynomial basis

($\Psi_l(t)$), and (b) Fourier basis ($\Psi_f(t)$) are analysed, in order to simplify the weight learning for the analysis; they are well known approximators used for curve fitting. They are given by,

$$\begin{aligned} \Psi_l(t) &= \sum_{i=0}^{n} a_i P_i((2t - t_d)/t_d), \\ \Psi_f(t) &= a_0 + \sum_{i=1}^{n} a_i \sin(i\omega t) + b_i \cos(i\omega t), \end{aligned} \tag{6.5}$$

respectively, where t_d is the duration of the movement and the corresponding weights are thus given by $\hat{W}_l = [a_0, \ldots a_n]$, and $\hat{W}_f = [a_0, a_1, \ldots a_n, b_1, \ldots b_n]$. The Legendre polynomials were computed using the standard Rodriquez formula; since the polynomials are defined in $[-1, +1]$, they are shifted to accommodate the entire duration of the intended movement. These synergies have another convenient property that their magnitudes are bounded, i.e. abs($\Psi(t)$) ≤ 1. This property is essential for nonlinear TSDA using empirical balancing since the method involves perturbing the inputs using unit impulse signals [Lall and Marsden, 2002].

6.3.1 Trajectory Specific Dimensionality Analysis

The TSDA framework was used to compare a set of benchmark trajectories on the Tethered Mass system; each trajectory describing a motion from the origin to an end position $[0.5, 0.5]$ in cartesian space.The trajectories, seen Fig.6.2a, were chosen to illustrate the differences in dimensionality, and were obtained by specifying via-points. The appropriate weights were then computed and the dimensionality analyed. The results of the dimensionality analysis are shown in Fig.6.2b. As it can be seen, each of the trajectories results in a variation in the dimensionality. Furthermore, the obtained reduced dimensionality $D_{\hat{W}}$ depends on the appropriate choice of basis, although the intrinsic reduced dimensionality $D_I = 1$. It can however be seen that, the minimum dimensionaltity is obtained for the straight line trajectory T_1 irrespective of the basis. The lower bar chart in Fig. 6.2b,d is the dimensionality cost formulated in this paper based on HSVs. This kind of cost enables the numerical computation of MD control using existing routines, since this form of cost function is continous.

6.3.2 Minimum Dimensional Control

MDC was synthesised for the tethered mass system as shown in Fig. 6.2c,d. MDC was synthesised to generate the optimal trajectories to reach position $0.5m$ away along the diagonal in $0.8, 1.0$ and 1.2 seconds with zero initial and final velocity, using (1) Fourier basis, and (2) Legendre polynomial basis. Minimisation of the dimensionality cost obtained sigmoidal straight-line trajectories with bell-shaped velocity profiles. There are minor differences between Fourier and Legendre polynomial bases in the velocity profiles, in that the peak velocity correspond to the Minimum Acceleration model in the former and the Minimum Jerk model in the latter. This might be because of the time profile of the bases. As it can be seen, the reduced dimensionality of a given trajectory is dependent on the synergy. However, the straight line path is the minimum dimensional trajectory independent of basis.

The MDC experiment was repeated on the kinematic chain system for a set of reaching targets within the workspace of the arm. Similar to the linear case, the constrained minimisation was initiated with zero velocity at the boundaries. A constraint tolerance of $\epsilon = 10^{-2}$ was used as a terminal criterion for the minimisation.

The trajectories resulting from MDC can be seen in Fig.6.4 for the polynomial basis synergies. Smooth sigmoidal near-straight line trajectories emerge for some movement durations; the results are presented for $t_d = 2.5s$ and $t_d = 3.0s$. The time normalised velocity profiles are again bell shaped and nearly symmetric, with the peak velocity in each axis dependent on the movement amplitude. In the kinematic chain case, the correspondence to Minimum Acceleration (MA) trajectories [Ben-Itzhak and Karniel, 2008] is closer (black dashed lines in Fig. 6.4a,b). The similarity of the obtained results to human movements suggests that the MDC criterion might represent an alternative perspective on the principles limb movement control in humans.

6.4 Summary and Conclusions

In this study the relationship of the reduced dimensionality to the input of a system is examined under the perspective of the muscle synergy hypothesis. Muscle synergies are defined as coordinated activations of muscles. It has been suggested that the mechanism employed by the CNS in simplifying the control problem is to modularise the control input by grouping together muscle activations; spatio-temporal regularities observed in EMG and kinematic data of human movements are cited as evidence (see Sec. 2.2.6).

In particular, the temporal synergy model consists of a set of task-independent basis patterns and the tasks are specified by the weight matrix linearly combining the synergies. The muscle synergy hypothesis contends that the motor coordination problem is directly simplified through input dimensionality reduction. Although the synergy hypothesis explains a number of observed features in the inter-muscle coordination in movement, it has faced criticism for being a phenomenological model of observed behaviour. A bottom-up approach to examining the reduced dimensionality in behaviour due to employment of synergies can provide testable conditions for the validation of this hypothesis. This study provides two key methods towards quantifying this relationship : Trajectory Specific Dimensionality Analysis (TSDA) and Minimum Dimensional Control (MDC).

The insight underlying this study is that under synergistic control, the behaviour of the system can be described by the dynamics of an equivalent system which incorporates the trajectory in the form of the weight matrix. A constrained-reformulation of the system using a given combination of synergies and then quantifying its reduced dimensionality therefore allows the comparison of the dimensionality of behaviour. This quantification is thus a task and synergy specific measure of the reduced dimensionality of a system. A measure using Hankel Singular Values is employed for quantifying the reduced dimensionality. The experiments show how this method can be used to compare various trajectories in terms of dimensionality, i.e complexity of generating the control.

The second outcome of this study is the extension of the TSDA framework from an optimisation perspective, the proposed Minimum Dimensional Control (MDC) model. This method computes the optimal dimensional trajectory (and the corresponding weights) for a given system utilising a specified set of synergies for accomplishing a task. Using a HSV measure of reduced dimensionality, a real-number valued cost function is derived for minimum dimensionality. The experimental results show that the numerical optimisation of this cost for a reaching task resulted in sigmoidal straight line trajectories with symmetric bell shaped velocity profiles as the minimum dimensional solution. The results were replicated in both a linear and a nonlinear model system and were tested on a synergy defined by a set of polynomials. This result constitutes a key outcome of this study and this thesis since it is indicative of reduced dimensionality principles underlying motor control.

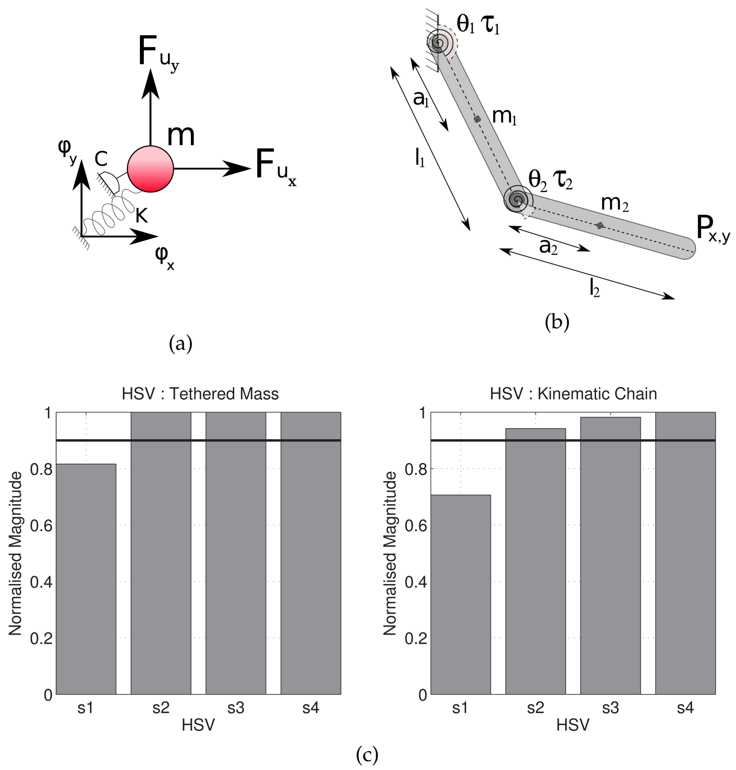


Figure 6.1: Physical systems employed for demonstrating the TSDA; (a) Tethered mass (Linear) : Motion of the mass is constrained to a 2D plane. The mass is anchored to the origin by weak passive forces and actuator forces are applied in two orthogonal directions. (b) Two-link planar compliant kinematic chain (Nonlinear) : End-point motion is constrained to a 2D surface. Passive compliance and damping forces act on the joints and joint torques are used to actuate the system. The state space descriptions of these systems have identical input (2), state (4) and output (2) dimensionality. (c) Comparison of the intrinsic dimensionality reduction for these two systems; the choice of threshold of $t_r = 0.9$ (black solid line) results in state-space reduction to dimensionality $\mathcal{K} = 1$ for both systems.

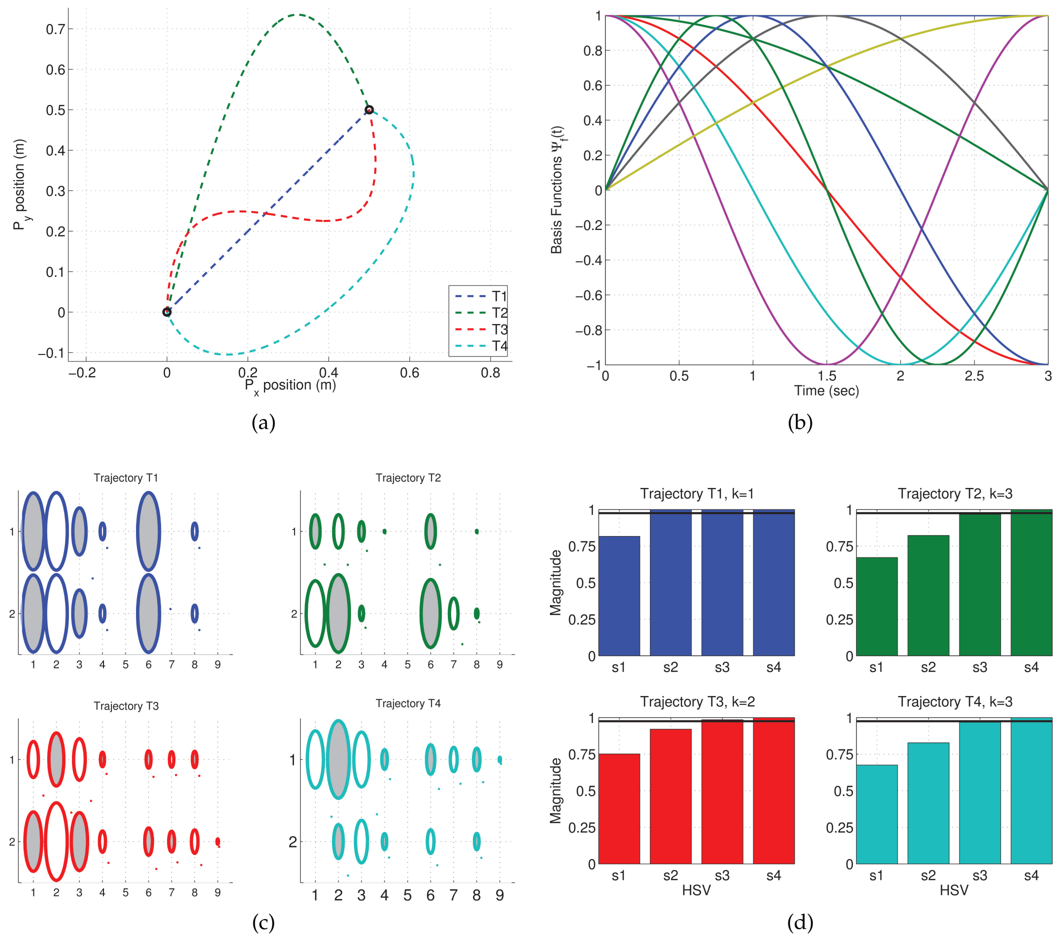


Figure 6.2: Trajectory Specific Dimensionality Analysis (TSDA) computed on Fourier basis synergies (order 4) actuating the tethered mass system; the task is to reach position $\phi_d = [0.5, 0.5]^t$ in a time span of $3.5sec$. (a) The four benchmark trajectories $[T_1, \ldots T_4]$ followed by the system under synergy control; (b) The Fourier basis temporal synergies used to compute control; (c) The weight matrix computed through a least-square procedure and inverse dynamics - represented as a Hinton diagram (ellipse size is the magnitude, a dark region denotes positive weight and white region denotes a negative weight). Note that the size of weight matrix is 2×9, with the rows corresponding to the 9 components necessary for a 4^{th} order Fourier basis; (d) The normalised HSV magnitudes for the reformulated composite systems for each trajectory. For a threshold magnitude choice of $t_r = 0.975$, the reformulated systems result in a DR of $\mathcal{K} = [1, 3, 2, 3]$. The system corresponding to the straight line trajectory T_1 is minimum dimensional as measured by the HSV magnitudes.

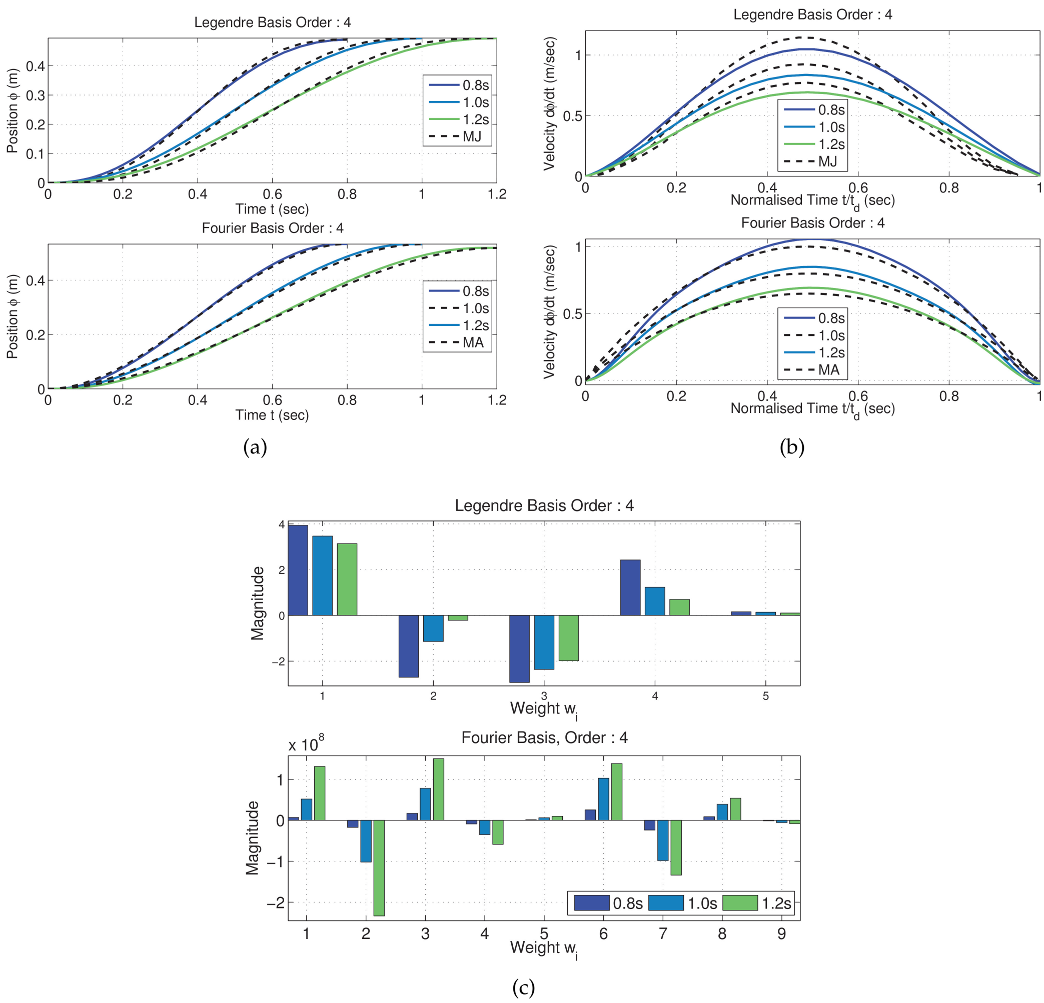


Figure 6.3: Minimum Dimensional Control on the Tethered Mass for reaching position $\phi_d = [0.5, 0.5]^T$ from the origin - 2 kinds of synergies (Legendre basis of order 6 and Fourier basis of order 4) and 3 desired time spans ($t_d = [0.8, 1.0, 1.2]$) analysed. Trajectory of mass traces sigmoids for all time spans and for both kinds of synergies. Trajectories (a) are similar to the Minimum Jerk (MJ) criterion for the Legendre polynomial basis and Minimum Accelaration (MA) for the Fourier basis case; (b) The corresponding bell-shaped symmetric velocity profiles. Optimum dimensional weights (d) for both Legendre polynomial and Fourier basis synergies linearly changes with increase in movement duration for each actuator.

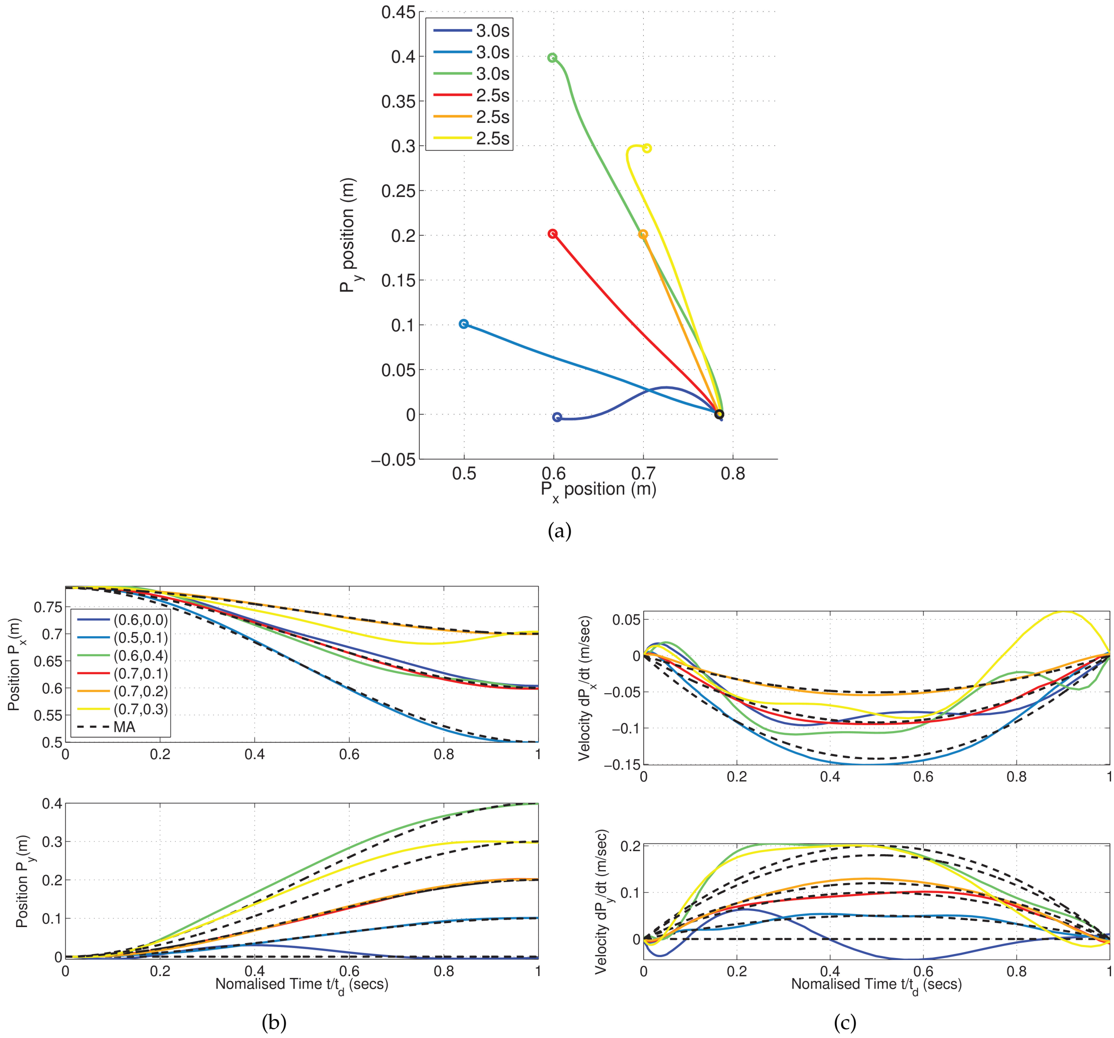


Figure 6.4: Minimum Dimensional Control on the Kinematic Chain for reaching various positions using Legendre basis synergy (order 7). Results show feasible MDC results for targets $[0.6, 0.0]$, $[0.5, 0.1]$, and $[0.6, 0.4]$ in a duration of 3.0 secs, and targets $[0.7, 0.2]$, $[0.7, 0.3]$, and $[0.6, 0.2]$ in a duration of 3.0 secs. Trajectory of endpoint is sigmoidal (a)velocity profiles show skewed bell shapes (b). The peaks of the velocity profiles however are close match to the Minimum Acceleration (MA) criterion result.

Chapter 7

Dynamical Movement Primitives and Reduced Dimensionality

This chapter reports on work presented in [Kuppuswamy et al., 2013] which can be found in Appendix E. This paper investigates the research question :

4. How can reduced dimensionality be exploited for coordinated movement control using dynamical movement primitives?

Kuppuswamy, N. , Ajallooeian, M., and Hauser, H. (2013). **Dynamical Movement Primitives and Reduced Dimensionality** (to be submitted shortly).

Abstract : *Elucidating the mechanisms underlying motor coordination in the realisation of movements is important for both understanding natural behaviour and control design of artificial systems. Dynamical Movement Primitives have been proposed in this context as a control architecture and modelling tool composed of a programmable pattern generators to encode and replay trajectories. This paper presents an analysis of the dimensionality reduction properties of the DMP whilst controlling a linear system. First, a task-specific reformulation of the controlled system. The dimensionality of the resulting reformulated system is then analysed using Hankel Singular Values, and reduced dimensional controllers are synthesised using the technique of linear balancing. The method is empirically first tested using a simulated system of a chain of mass-spring damper elements, the results show that there is an increase in the percentage of reduction with increase in the dimensionality of the system under control; the reduced dimensional models synthesised compare suitably to the full dimensional models in control performance. The task specific formulation and its dimensionality reduction allows comparison of tasks (trajectories) in terms of dimensionality; this is demonstrated with three kinds of benchmark trajectories. The approach is then applied to the problem of trajectory planning in the leg of the Cheetah quadrupedal robot. The approach shows how some trajectories could allow usage of lower dimensional models. The results show that the DMP can be an effective tool for not only encoding movements but also for decreasing the dimensionality of the controlled system apart from comparing trajectories.*

7.1 DMP Architecture

DMPs are a recently developed novel control architecture which successfully combine the principles of dynamical systems with an optimal learning. They are ideally suited for encoding movements on high DoF morphologies, for eg. whole body humanoid behaviours (See 2.3.5).

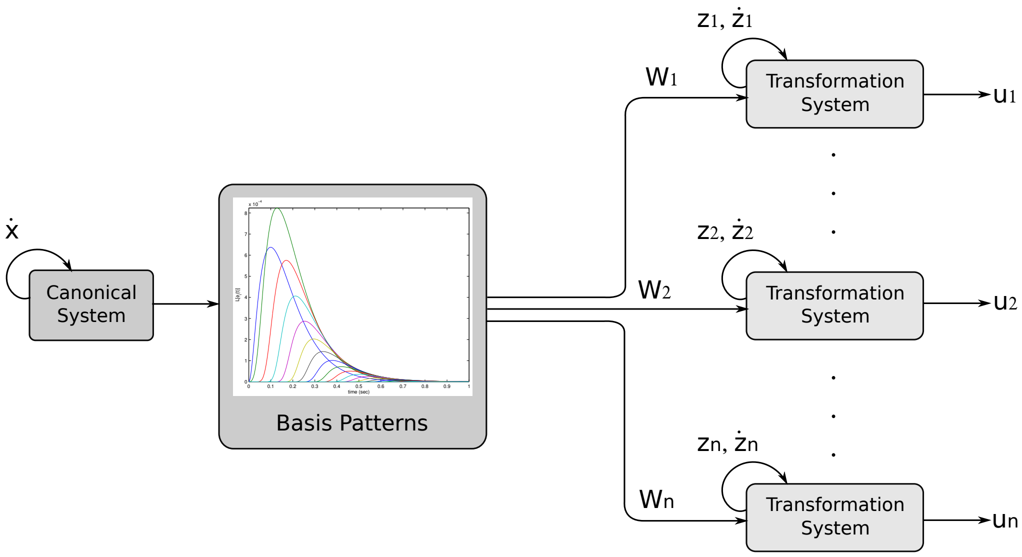


Figure 7.1: Architecture of the DMP; control commands are generetaed by a set of n transformation systems, which are dynamical systems with well understood characteristics are perturbed by a weighted combination of basis functions. The coupling between DoF of input is achieved through utilising a common canonical system - a simple first order system that acts as a virtual time for the duration of a movement

As described in Fig. 7.1, a system of n DMPs (to control n DoF) is described by n 2^{nd} order forced and damped harmonic oscillators interconnected by the state variable χ acting as virtual time variable, as described in the following set of equations,

$$\tau \ddot{z}_j = \alpha_z(\beta_z(g_j - z_j) - \dot{z}_j) + f_j(\chi), \qquad \tau\dot{\chi} = -\alpha_\chi \chi, \tag{7.1}$$

where, the output of the j^{th} DMP is z_j and α_z, β_z, α_χ are task independent constants, g_j is the goal position. The time constant τ is used to modulate the duration of the learned trajectories. The forcing function $f_j(x)$ is defined by a weighted and normalized summation of Gaussian basis functions,

$$f_j(\chi) = \frac{\Sigma_{i=1}^N \Psi_i(\chi) w_{ij}}{\Sigma_{i=1}^N \Psi_i(\chi)} \chi(g_j - z_{0j}),$$
$$\Psi_i(\chi) = \exp\left(-\frac{1}{2\sigma_i^2}(\chi - c_i)^2\right),$$

where, the constants σ_i, and c_i are chosen to appropriately distribute the basis function over the entire trajectory. In this formulation, the training process aims to obtain the appropriate values of w_{ij} to suitably mimic a desired trajectory. Locally weighted regression [Schaal and Atkeson, 1998] was employed for learning the weights.

7.2 Methods and Results

In order to apply TSDA on the DMP system, the input must be described in the form of,

$$u(t) = \hat{W}\Psi(t), \tag{7.2}$$

this entails obtaining the component basis patterns by which movements are composed using the DMP. This was carried out by simply solving the DMP analytically. This solution however is not describable in the form of simple known analytical expressions and therefore the alternative method employed is to extract the component basis patterns computationally. The proposed method for this process is named Iterative Basis Extraction (IBE). The basis from the DMP extracted through IBE is depicted in Fig. 7.2.

In the analysis reported in Appendix E, simulation results are presented for linear and non-linear systems. The linear system results are briefly summarised here. The system of a chain of mass-spring-damper is utilised for the experiments similar to the model reported in Chap. 4. The bases synthesised using IBE are then utilised to train some benchmark trajectories. Three such trajectories are reported,

1. exponential-cosine functions (damped cosine)
2. 2^{nd} order polynomials
3. 5^{th} order minimum jerk polynomials

In each of these cases, the TSDA methodology is applied to compute the reduced dimensionality. This process is then repeated for various increasing chain-lenghts. Balanced reduction is employed on this system – this is done using the matlab routines of *balred* since analytical formulations exist for the linear system case. The quantification of the reduced dimensional measure $\mathcal{D}$ (see Chap. 3) utilises a threshold on the normalised Hankel Singular Values (HSV).

The variation of the reduced dimensionality with the increase in chain length, i.e. dimensionality of the full-dimensional system is shown in Fig. 7.3a. The results show that in some trajectories there is a greater reduction in dimensionality than others; the expo-cosine results in greater reduction in than the others utilised in this benchmark. Furthermore, the percentage reduction of utilising some trajectories also seems to increase with increase in dimensionality of the mechanical system. In all cases however, the composite system under DMP control is always lower in dimensions to a reduction purely on the mechanical component; this indicates that the DMP always contributed to reduced dimensional behaviour of the system.

In this analysis it is not sufficient in just comparing dimensionality measures and the effect of the task-specific dimensionality reduction needs analysis. The comparison of trajectories generated using the full and reduced dimensional systems is shown in Fig 7.3b. The comparison indicates that this approach can be used for both synthesis of task specific reduced order models, and the models themselves are accurate enough to be employed for purposes of behaviour prediction or planning.

7.3 Summary and Conclusions

In this study, a method for extending the TSDA analysis proposed in Chap. 6 was presented quantify the relationship between reduced dimensionality $\mathcal{D}$ and the DMP. The DMP control strategy has increasingly found use in the control of high dimensional robots from the viewpoit of imitation learning from human motions. The quantification presented in this study could be used

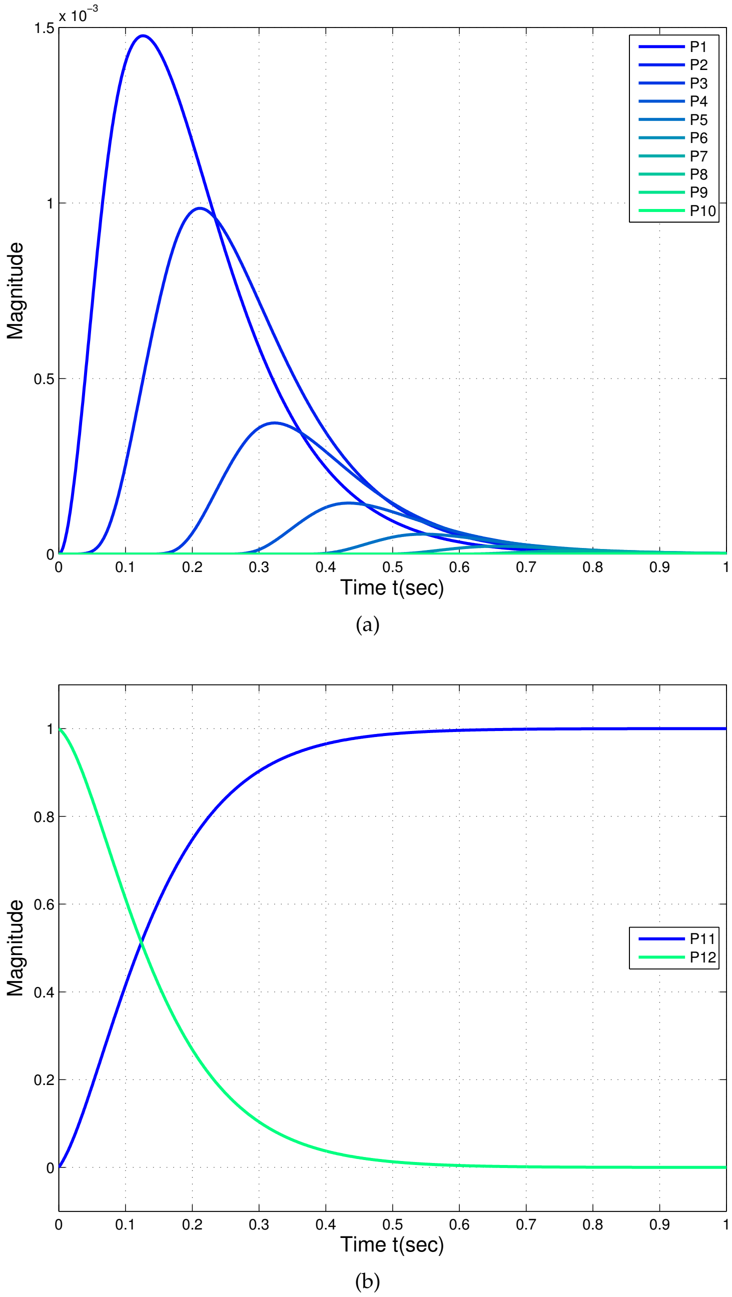
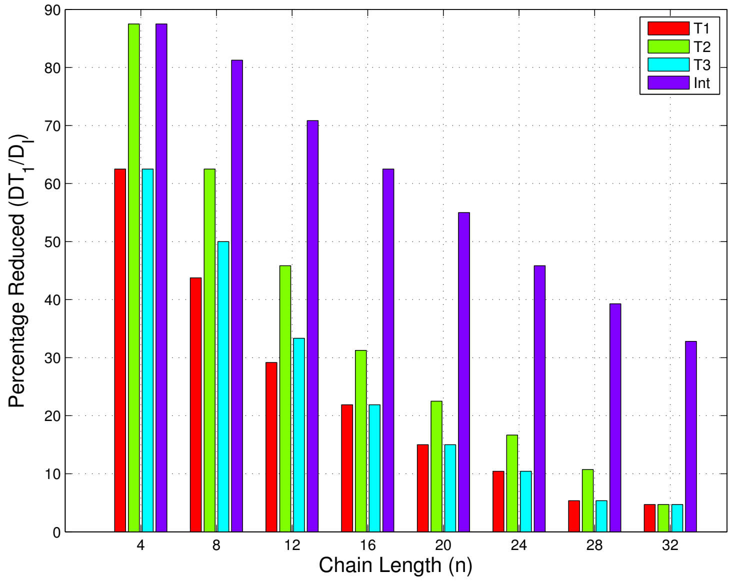


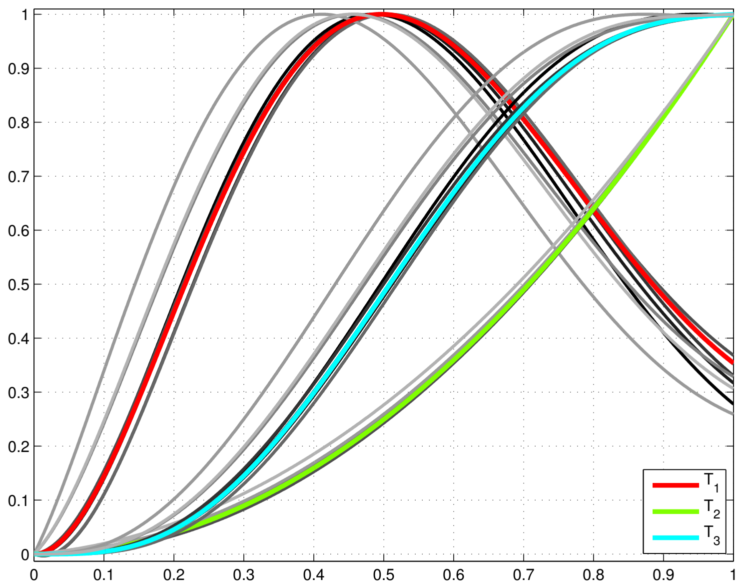
Figure 7.2: Bases of the DMP obtained through the Iterative Basis Extraction (IBE) procedure : (a) first 10 basis patterns of the DMP due to the Gaussian basis functions , (b) the last 2 basis patterns of the DMP due to the transformation system.

in conjunction with existing online learning approaches for training the DMP. This approach is a pathway to high-speed online adaptive DMP learning for high-dimensional robots.

In this study, this problem is treated by the proposal of a Iterative Basis Extraction (IBE) method. This transforms the DMP controlled system into the temporal synergy formulation of



(a)



(b)

Figure 7.3: Trajectory Specific Dimensionality Analysis (TSDA) on the DMP; (a) decrease in reduced dimensionality with increase in chain length (b) trajectories of the full and reduced dimensional systems in replicating the three benchmark tasks : damped exponential cosine (T1), 2^{nd} order polynomial (T2), and minimum jerk 5^{th} order polynomial (T3). The trajectories of the equivalent reduced dimensional systems is depicted in the shades grey, with darker shades representing a greater reduced dimensionality.

the Chap. 6. TSDA can then be applied on the system of newly extracted bases when applied to control any predefined mechanical system.

The results show that some trajectories result in a greater degree of reduced dimensionality than others. Furthermore, the degree of decrease increases with the dimensionality of the original system. For a given mechanical system using DMPs some trajectories are seemingly simpler to model than others. Task specific reduced dimensional models which best capture behaviour can then be synthesised. Towards a more realistic case-study, comparison of leg trajectories for a simulated quadrupedal robot was demonstrated for stepping behaviours. The outcome of this study is the speeding up of the control learning for biomimetic robot application through the exploitation of reduced dimensionality.

Chapter 8

Dimensional Change and Development

This chapter summarises the research reported in [Kuppuswamy2013c] which can be found in Appendix F. The paper investigates the research question :

4.a. How can dimensional change in a developmental perspective be achieved through dimensionality reduction?

Kuppuswamy, N. , Oesinghaus, J., and Harris, C. M. (2013). **Development and Dimension Reduction** (to be submitted shortly).

Abstract : *One of the fundamental problems in developmental robotics relates to the progressive spontaneous acquisition of motor abilities by an organism. Throughout this process, the speed of aquiring abilities, which we term 'learnability', is strongly limited by the dimensionality of the sensori-motor space; this in turn could affect the survival of an organism. The existing theories on motor learning have been strongly influenced by the Bernstein notion of dimensional increase accompanying development, although counter proposals have also been suggested. The problem of redundancy resolution has also been tackled from the perspective of optimal control of motor behaviour and through theories of motor primitives, although the relationship to development is not yet clear. In this work, we present a formalisation of the dimensional change problem from the perspective of control dimensionality reduction. By utlising a projection of the neuromuscular dynamics into lower dimensional subspaces quantified by a measure called Hankel singular values, we demonstrate theoretically that a progressive acquisition of skills of increasing complexity can be achieved; the change in dimensionality induced through changes both in the natural dynamics, and in the task space. As a case study, we present empirical results on dimensional change in reaching and manipulation task using a simple kinematic chain system modelling arms; the growth process resulting in gradual morphological and material property changes. The results show that there could be an optimal "path" in parameter space wherein the growth can regulate the learnability.*

Developmental plasticity is concerned with the irreversible changes that the newborn phenotype undergoes as it matures and develops, which typically takes a considerable fraction of the phenotype's lifetime. In either case, it seems likely that there should be a premium on learning as quickly as possible for a given level of competence and task complexity. The speed of learning, which can be termed'learnability', depends ultimately on the dimensionality of the behavioural task space, and many have argued that dimensionality may be manipulated during development to improve learnability (see Sec.2.2.1). Of course, any control over dimensionality must be manifest in the neural and/or structural architecture of the organisms, and must be inheritable itself (i.e. coded in DNA).

8.1 Results and Methods

The approach presented in this work is to formalise the problem of dimensional change by utilising the framework of control dimensionality reduction. Using a notation similar to that of Chap. 3, the definition of a dimensionality measure $\mathcal{D}$, computed on a system $\mathcal{F}$ is utilised along with formal definitions of what is a task T in order to compute a control action $u(t)$. The theoretical formulation allows posing of the dimensional problem as one of progressive increase in task complexity – this is quantified through a progressive decrease in a space of possible inputs which can solve the task.

The proposed mathematical framework was then used to demonstrate how dimensional change can be achieved in a simulated system of an arm (see Chap. 5) through parametric changes in the natural dynamics, i.e. $f(x)$.

The experiment aimed to demonstrate that learnability can be controlled through a variation in passive properties – thus mimicking the growth process in nature. For the results presented in Fig. 8.1, the primary parameter under investigation is that of joint-damping; this is chosen taking inspiration from prior results in linear systems presented in Chap. 4. The parameter settings for simulation were identical to the system presented in Chap. 5. Joint damping was varied within a range of of $[0.01, 2.5]$ units. In each case, reduced dimensional models were synthesised using the method of empirical gramians. The variation in Hankel Singular Values (HSV) with change of damping can be seen in Fig. 8.1a. If the approach of threshold normalised HSVs is employed to synthesise reduced dimensional models, dimensional variation willl naturally result, due to the shift of the HSVs.

However in this case, the relationship to the task space is also to be demonstrated. Thus, each case, reduced dimensional models are synthesised of dimensionality $\mathcal{D} = [2 \dots 10]$ where the source system dimensionality is $N = 10$. Using each of these models, the step response is examined in relation to the full dimensional model. This allows the quantification of the error due to the reduction; the Cartesian distance between the reduced and full dimensional models at the end of a fixed 5 second interval from movement initiation is utilised. Fig. 8.1b shows the results of this experiment, where the error is plotted as a function of both dimensionality and damping. The results show that damping changes in some directions can result in dimensional increase while progressively decreasing control and prediction error; it is assumed that the difference between reduced and full dimensional representations captures the progressive change in control and prediction errors.

8.2 Summary and Conclusions

In this study, the developmental theories of DoF increase/decrease during acquisition of motor skills in organisms is formalised as a problem of reduced dimensionality change. Specifically, changes in the natural dynamics and their impact on progressive task complexity increase are

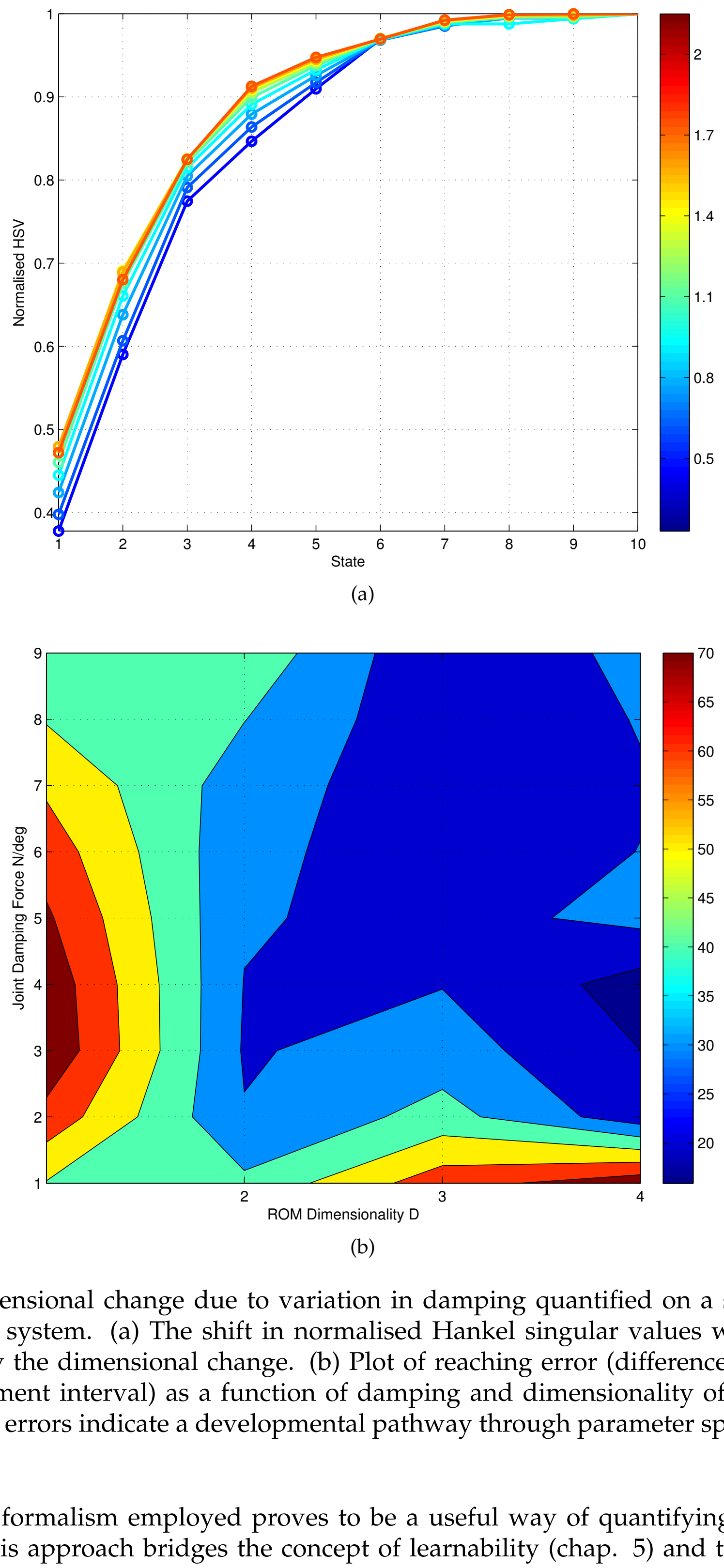


Figure 8.1: Dimensional change due to variation in damping quantified on a simulated 2 link kinematic chain system. (a) The shift in normalised Hankel singular values with damping, is used to quantify the dimensional change. (b) Plot of reaching error (difference in end position at end of movement interval) as a function of damping and dimensionality of the controller – regions of lower errors indicate a developmental pathway through parameter space

examined. The formalism employed proves to be a useful way of quantifying developmental phenomena. This approach bridges the concept of learnability (chap. 5) and the physiological process of maturation. This study formalises the results presented in chap. 4 from a task-space perspective.

The simulation results show that some forms of passive mechanical property changes can directly impact task complexity increase. This is enabled by an increase in the reduced dimensionality. The '*growth*' of passive mechanical properties is presented as a trajectory in a parametric space resulting in motor skill changes. The improvement of skills resulting from dimensional change, implies that optimal growth strategies exist within this parametric space. These results thus demonstrate how the growth process itself can regulate learnability, i.e. the reduced dimensionality $\mathcal{D}$.

From the perspective of developmental robotics, this approach can be used to exploit morphological changes for progressive skill acquisition. Technologies such as variable compliance can be used to directly enable the autonomous acquisition of various motor skills.

Chapter 9

Discussion, future work and conclusions

This chapter summarises the key contributions of this thesis and presents discussions on the outlook from multiple perspectives. The limitations of the presented methods are also discussed, along with the scope for future research.

From an optimisation viewpoint, in any discussion on problems in machine learning, and statistics the *curse of dimensionality* is inevitably encountered; this phenomenon requires careful consideration. Some commonly cited predicaments include sparsity of information, combinatorial explosion, the necessity for large number of samples and the intractability of computing solutions that comply with real-time requirements. A huge amount of effort has been invested in synthesising techniques for dealing with high-dimensionality.

From a control engineering perspective, high-dimensionality has also been a limiting factor. High-dimensionality complicates the synthesis of accurate models that can be used for control design.The high-dimensionality itself can be advantageous due to the flexibility it offers in dealing with novel contexts – humanoid robots are a prime example. Finding an approach that can mitigate the drawbacks of high-dimensionality is crucial in harnessing the full capabilities of a system.

Dimensionality reduction techniques present an ideal approach for circumventing some of these problems. As surveyed in Sec. 2.4.1, model dimensionality reduction methods have successfully been applied in many types of problems in simulation and control of complex systems such as integrated circuits, process plants etc. These methods are largely under-utilised in robotics and AI research. Finding principles for the exploitation of dimensionality reduction can lead to novel methods for high-dimensional robots.

In particular, in the field of study of Embodied AI, attempts have been made to identify various *design principles* which could lead to the manifestation of intelligent behaviour [Pfeifer and Scheier, 2001]. These design principles are often based on observations of biological systems and can lead to novel techniques for engineering.

In this vein, the primary contribution of this thesis is to demonstrate how reduced dimensionality can be exploited as design principle. This is accomplished through a systematic analysis of contributing factors to reduced dimensionality. The research here presented, was carried out in the form of five case studies, each of which examined a different approach to exploiting the reduced dimensionality of systems, as summarised in Table 9.1. Both theoretical and empirical demonstrations were employed to show how reduced dimensional behaviour can be taken advantage of effectively.

The methods proposed and the results demonstrated in thesis are aimed at applications in two inter-related target disciplines : *(a)* biological motor control and its development, and *(2)* methods

Case Studies	Research Focus	Test Platform (Simulated)
Natural Dynamics	Passive properties (stiffness, damping and mass)	Linear n-element mass spring damper chain
	Reduced Dimensional Control (motor primitives)	Linear model of pendulum robot
Task Space	Cartesian end position of limbs	Dynamic model of vertebrate limb with 2 joints and 1^{st} order muscles
Muscle Synergies	Trajectory Specific Dimensionality	Linear tethered mass
		Kinematic chain with joint compliance
Dynamical Movement Primitives	Trajectory Specific Dimensionality	Linear n-element mass spring damper chain
		Compliant planar robot let model
Dimensional Change	Progressive dimensional change and role of passive properties (stiffness, damping and mass)	Dynamic model of vertebrate limb with 2 joints and 1^{st} order muscles

Table 9.1: Studies carried out in this thesis and the corresponding simulation framework employed

for control of biomimetic robots. The principal outlook of this thesis is summarised next from the viewpoints of biology and robotics.

9.1 Outlook for biology

The biological viewpoint on reduced dimensionality, as surveyed in Sec. 2.2, encompasses many notions; there are a plethora of theories purporting to explain the mechanisms underlying the control of coordinated movement by the CNS. The problem of coping with high-dimensionality in neuro-mechanical substrate remains a major focus in many of the motor control theories. Although many hypotheses suggest that dimensionality reduction might be taking place, the exact mechanisms are still elusive.

In this context, control dimensionality reduction methods can serve as ideal tools for examining these unresolved questions from a synthetic viewpoint. In particular the contributions of this thesis can serve as a framework for unifying the contemporary approaches towards motor control theories and theories of ontogenetic development of motor skills.

9.1.1 Implications for the muscle synergy hypothesis

The contributions presented in Chap. 6 and Chap. 7 in particular are very relevant to the analysis of modular motor control theories. In particular the proposed method of Trajectory Specific Dimensionality Analysis (TSDA) offers a framework by which the muscle synergy hypothesis can be examined in greater detail. One of the primary criticisms levied against the muscle synergy hypothesis is concerning the phenomenological nature of the results. The TSDA coupled with accurate bio-mechanical models can be utilised to test the involvement of synergies in various voluntary movements. This also validates the premise that modular control strategies simplify learning by reducing dimensionality.

The Minimum Dimensional Control (MDC) proposal in particular relates modular motor control to existing work on optimisation approaches. The synergy-independent nature of the trajectories synthesised by MDC suggest that dimensionality reduction of the behaviour underlies the learning and adaptation of movements; the cost of adaptation is related to the cost of computing and predicting behaviour – i.e. the reduced dimensionality.

9.1.2 Implications for development

The Bernstein notions of DoF increase through the three-stage learning model has been influential in developmental motor neuroscience. Counterproposals have questioned the nature of this dimensionality change from a dynamical systems perspective (see Sec. 2.2.1). Developmental theories of motor control have until now been limited by the insufficiency of conclusive experimental evidence; the difficulties in performing behavioural studies with infants is a prime factor.

This thesis has some fundamental implications for development since it presents a quantification of the progressive and gradual growth in complexity of abilities and behaviour through the framework of dimensionality reduction as presented in Chap. 3

In Chap. 5 and Chap. 8 the concept of *learnability* is introduced as a limit on the rate of learning that can take place in an organism during development; this is dictated by the dimensionality. The employment of a reduced dimensionality principles in the learning can result in significant advantages, potentially impacting survival and thus fitness of an organism.

A key contribution of this thesis towards development is presented in Chap. 8. The developmental notion of dimensional change is formalised from the perspective of reduced dimensionality. This could potentially lead to the generation of testable hypotheses for developmental theories. In particular, the suggestion of physical growth itself regulating the learnability seems promising.

9.2 Outlook for robotics

One of the important consequences of embodied AI has been the huge spurt in interest in bio-inspiration and bio-mimetics (see Sec. 2.3). This has lead to research in identifying the fundamental organisation principles through which nature seems to evolve seemingly simple solutions to solve complex problems – underactuated systems such a passive dynamic walkers are a prime example [Collins et al., 2005]. However, biomimesis has also resulted in robotic morphologies at the other end of the spectrum in terms of complexity - anthropomimetic robots such as the ECCE I are good examples of this design philosophy [Wittmeier et al., 2013]. The resultant complexity has meant that design of such robots has so far focused chiefly biological modelling. They serve as good test platform for synthetic approaches to understand the richness of motor skills in nature. From an application viewpoint, the behavioural diversity that is enabled by these morphologies is potentially of great value. Although the high-dimensionality remains a fundamental limiting factor on the applicability of such systems.

9.2.1 Implications for model-based control

One of the principle motivations for the research presented in this thesis is that reduced dimensionality renders model-based approaches tractable. Traditional robot control methods such as feed-forward control, optimal control, or adaptive control can thus potentially be applied on complex robot morphologies.

In particular, the study presented in Chap. 4 is relevant to the synthesis of simplified models of complex systems. The results show how passive properties in a system can be exploited to regulate model complexity due to reduced dimensionality.

The method of progressive dimension change demonstrated in Chap. 5 shows how model complexity for robotic systems can be progressively scaled to match task complexity requirements.

9.2.2 Implications for motor/movement primitives for robots

Bio-inspired control techniques such as the Central Pattern Generators (CPG) and Dynamic Movement Primitives are aimed at resolving the high-dimensional coordination problem (see Sec. 2.3.4). They have so far only been applied to rigid robots with well defined model based controllers.

The study in Chap. 4 is relevant to the control of compliant and tendon-driven robots. The proposed method is used to demonstrate how a motor primitive inspired control architecture can be self-acquired by a complex robot and can lead to simplification of the control.

In the case of the DMP (see Sec. 2.3.5) diverse applications have been demonstrated in humanoid systems. However, DMPs have only been used for encoding kinematic trajectories for robots with well tuned low-level controllers [Kuppuswamy and Alessandro, 2011].

The study presented in Chap. 7 is directly relevant to the analysis of dimensionality of systems under DMP control. The results of this study are important for two kinds of applications : (a) usage of DMPs in planning and encoding dynamic trajectories, especially for complex robots, and (b) usage of DMP for adaptive control, such as [Buchli et al., 2011]. The proposed method enables task-specific reduced dimensional models to be used for expediting control learning and adaptation.

9.2.3 Implications for developmental robotics

Cognitive developmental robotics aims at incorporation of principles underlying ontogenetic development [Asada et al., 2001]. This approach can also be used as a synthetic methodology for developmental phenomena.

The methods proposed in this thesis are highly relevant to development of motor abilities in complex artificial systems. The study presented in Chap. 5 directly addressed the developmental issue of DoF change in the sense of [Bernstein, 1967]. This holds implications for developmental methods of progressive increase in dimensionality of the sensorimotor space through DoF freezing and unfreezing [Berthouze and Lungarella, 2004]. The method proposed in this thesis can achieve the necessary dimension change without requiring these kinematic constraints during development.

The method presented in Chap. 8 demonstrated a relationship between dimensional change and passive properties. This result has important implications developmental robotics. This approach potentially allows the realisation of a growing robotic organism which can progressively regulate its development of mental abilities.

9.3 Limitations and Scope for Future Work

This thesis demonstrated various methods for exploiting reduced dimensionality in the design and control. There is a lot of potential for future work in extending these results. From a robotics viewpoint, the various methods proposed in this thesis are aimed at real-world applications, particularly in the case of complex biomimetic robots such as ECCE I. However it remains to be seen if the proposed methods can cope with the large degree of nonlinearities present in such real-world systems. From a biological viewpoint, the proposals of TSDA, MDC and dimensional change all raise interesting questions; working towards testable hypotheses is a natural extension is necessary towards validating some of these claims.

Although this thesis implied that the rate of learning can be regulated through reduced dimensionality, this assertion remains to be tested using known learning methods. In particular, the suggestion for utilising variable compliance actuator mechanisms to simplify the motor-skill learning requires testing in real-world conditions.

In terms of algorithms, only reduction methods based on projection were used for reduced dimensionality analyis. This may not be sufficient for coping with all kinds of nonlinearities and needs further tests in the real-world.

9.4 Conclusions

This thesis presented systematic research into how the phenomenon of reduced dimensionality can be exploited in the design and control of embodied systems. An extensive review was first presented detailing how the problems of high-dimensionality have been traditionally been treated in biological motor control and in a developmental context. Relevant state of the art in robotics and in control theory was also reviewed and a case was presented for a unified framework in approaching this problem. Based on the literature review, a mathematical framework of reduced dimensionality analysis was proposed.

The first study demonstrated how natural dynamics can contribute towards reduced dimensionality. It was shown how appropriate configurations of passive properties can result in reduction of dimensionality of linear high-dimensional systems such as mass-spring-damper chain systems. Taking inspiration from biological theories of motor primitives a control architecture was proposed for compliant redundant robots. It exploits the natural dynamics and utilises developmental principles in its synthesis.

The second study demonstrated how the task space can be utilised for reduced dimensionality. The notion of learnability was presented as a limit on the rate of learning due to reduced dimensionality. The Bernstein notion of DoF increase accompanying development was quantified using the reduced dimensional modelling and control framework.

The third study demonstrated how reduced dimensional behaviour of the entire system can be influenced by utilising modular control strategies. A key result were the proposals for Trajectory Specific Dimensionality Analysis and the Minimum Dimensional Control. The simulation experiments applying the MDC proposal resulted in the emergence of behaviours with some of the invariant properties of human movements.

The fourth study extended the TSDA framework in quantifying the reduced dimensional behaviour of Dynamical Movement Primitives. The simulated results indicate that task-specific reduced dimensional models can be synthesised, and that some trajectories result in greater than others. The proposal was also extended to the nonlinear case and applied for the problem of trajectory planning for compliant robot legs.

The fifth study presented a mathematical framework for the quantification of the phenomenon of dimensional change. This notion which is highly relevant to the developmental process of acquisition of motor abilities of increasing complexity is analysed from the perspective of control dimensionality reduction. Various ways by which dimensional change can be achieved in systems are theoretically presented. The results on analysing passive properties indicates that there could be optimal paths in parameter space wherein the physical process of growth can regulate the learnability in systems. The results presented in this thesis have potential implications for both robotics and biology and this is discussed in detail.

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Appendix A

Effect of physical variation on the reduced dimensional control of a mass-spring-damper chain system

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Effect of Physical Variation on the Reduced Dimensional Control of a Mass-Spring-Damper Chain System

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***Abstract*—In this work the relationship between growth and ontogenetic development of motor control is studied from the perspective of reduced dimensionality in control; such motor control strategies have been suggested as a possible mechanism circumventing the degree of freedom coordination problem. The relationship between reduced dimensional behaviour and parametric variation is empirically analysed in a simulated actuated mass-spring-damper system, as a loose analogy to physical growth process in vertebrate limbs. The resultant dimensionality change is analysed and ideal directions for growth, in terms of physical parameter variations, are discussed.**

I. Introduction

While it is well recognised that the physical phenomenon of the growth process affects the ontogenetic development of motor abilities in organisms, the actual mechanisms are far from being understood. One of the key open questions concerns the relationship of growth to the *degrees of freedom coordination (DoF) problem* [2]; one proposed coping mechanism is to freeze and progressively unfreeze some of the DoF in parallel to growth and motor skill acquisition [3]. An alternative strategy for redundancy resolution is control based on a reduced dimensional representation of a system's behaviour. In this context, the theory of motor primitives (or muscle synergies) [1], [5], suggests a strategy enabling control dimensionality reduction through enforcement of linear constraints in the input. It has been proposed that the motor primitives may be obtained by computation using a reduced dimensional model of the musculo-skeletal dynamics [1]; the number of primitives required is simply the number of dimensions to which the system can be reduced to. However, dimensionality and therefore the reducibility of a mechanical system, is dependent on the physical parameters within the mechanical system.

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In this work, the effect of some parameter variations, i.e. of mass and damping, on the reduced dimensionality is empirically studied in a simulated system consisting of a $1D$ chain of 10 masses interconnected with linear springs and damping elements. The chain is fixed in the proximal end, and free to move in the distal end, as depicted in Fig.1 and each mass element can be independently actuated by applying forces. The system is perturbed and the resulting behaviour is used for dimensionality reduction using Proper Orthogonal Decomposition (POD); the change in dimensionality is analysed for various parameter values. Although a relatively simple model, we loosely compare it to vertebrate limbs and try to gain insight and generate testable hypothesis on the effect of physical growth on the dimensionality of control.

II. Experiments and Results

We investigate the effect of physical variations (growth of parameters) on reduced dimensionality for the following parameters:

1) Ratio of Damping to Stiffness (*Damping Ratio*) $D_{r_i} = c_i/k_i$: quantifies the local stiffness along the chain,
2) Ratio of Mass to Stiffness (*Mass Ratio*) $M_{r_i} = m_i/k_i$: quantifies the relative weight along the chain,

The stiffness is maintained constant throughout (at $1N/m$) and the parameter of interest p_i at position i (M_{r_i} or D_{r_i}) is subjected to a change $\Delta p_i = f(i, g)$, where g is an integer denoting the *growth* rate of the parameter. The growth rate could be thought of as a loose analogy to the effect of time on the natural growth process since growth seems to occurs in quanta [6].

The experiments which were carried out on an increasing growth g explored the following 2 Scenarios : a) Uniform Growth throughout an arm with equal initial distribution, $p_i = a + bg$; b) Uniform Growth throughout an distally distributed arm, i.e. $p_i = (a + bi)g$. The constants a and b are dependent on the range of variability of the parameter. Based upon these 2 scenarios, the experiments performed studied 2 kinds of growth effects, i) Damping ratio decrease; ii) Mass ratio increase.

For each run, the simulated system is perturbed by fixed duration pulse inputs. Then POD, (principal component analysis on the state trajectory, i.e. positions and velocities of the masses), is used to compute a reduced dimensional model (which can then be used for control), by truncating to the minimum set of normalised components below a threshold $t\%$.

For the first experiment, the damping ratio was decreased in 25 steps of $0.0196N/m/s$ starting from, $0.5N/m/s$ uniformly for scenario A, and a proximal to distal linear distribution in the range $[0.125, 0.01]N/m/s$ for scenario B. For the second experiment, the mass ratio was increased in 25 steps of $1kg$ starting from, $0.5N/m/s$ uniformly for scenario A, and a proximal to distal linear distribution in the range $[6.25, 1]kg$ for scenario B.

The results on dimensionality increase in Fig. 1 show a much greater dependency on damping ratio decrease (a), than on mass ratio increase (b). It is known damping tends to decrease dimensionality in mechanical systems, due to its effect of attenuating the higher frequencies in a system [4] the result in Fig.1a demonstrates a similar trend. On the other hand, while in principle, mass changes should not affect the dimensionality since it uniformly affects all the frequencies. However due to the effect of using a fixed time step for the POD analysis, a small increase in the dimensionality results, as can be seen in Fig.1b .

III. Discussion

Dimensionality reduction may be an essential component of the growth and development process of complex organisms and our framework allows us to test this relationship. Although the increase in mass during the growth process is a necessity, the results leads us to hypothesise that from a dimensionality and learning perspective, it might be beneficial to maintain a large degree of damping towards distal ends of limbs and progressively decrease it along with increase in the mental abilities, in order to optimally aid control development.

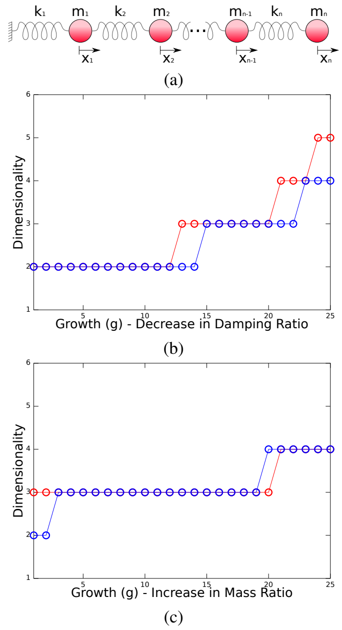

Fig. 1: Test System and Results (a) A $1D$ chain of masses interconnected with springs and damping elements. Reduced Dimensionality for Parametric Variations : (b) Damping ratio decrease, (c) Mass ratio increase, under scenarios of even (Red) and uneven (Blue) parameter distributions.

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Appendix B

Synthesising a Motor-Primitive Inspired Control Architecture for Redundant Compliant Robots

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Synthesising a Motor-Primitive Inspired Control Architecture for Redundant Compliant Robots*

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Abstract. This paper presents a control architecture for redundant and compliant robots inspired by the theory of biological motor primitives which are theorised to be the mechanism employed by the central nervous system in tackling the problem of redundancy in motor control. In our framework, inspired by self-organisational principles, the simulated robot is first perturbed by a form of spontaneous motor activity and the resulting state trajectory is utilised to reduce the control dimensionality using proper orthogonal decomposition. Motor primitives are then computed using a method based on singular value decomposition. Controllers for generating reduced dimensional commands to reach desired equilibrium positions in Cartesian space are then presented. The proposed architecture is successfully tested on a simulation of a compliant redundant robotic pendulum platform that uses antagonistically arranged series-elastic actuation.

1 Introduction

It has been argued that natural systems, in order to cope with uncertain, unstructured and dynamically changing environments evolve morphologies and material properties that are physically compliant (adaptable to external influences) and redundant (versatile in face of constraints), among other features [10]. The flip side of this argument is that the Central Nervous System (CNS) needs to cope with the large dimensionality thus induced. Even for simple end-point movements, a large number of muscles are recruited and, thus, have to be supplied with requisite input commands. Since the number of muscles is much higher than the number of variables in which the goal is defined, a single movement can be obtained by many different patterns of muscle activations; this is often referred to as Bernstein's *degrees of freedom problem* [3].

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This problem is also important from the point of view of designing robotic systems for the real world. The problem of controlling dynamically complex systems is typically approached by explicitly using the (*inverse*) kinematic or dynamical models of the plant. However, the computational complexity drastically increases with the number of degrees of freedom [6]. Learning and optimisation theory offer an alternative to solving the redundancy problem. However, optimization algorithms suffer from an exponential increment of the computational complexity as a function of the dimensionality of the search space, the so called "curse of dimensionality" [12], rendering them intractable. Unsupervised learning methods [13] have also been proposed in this context, however their scalability to complex redundant and compliant robotic systems is unknown.

An alternative paradigm would be to look for a reduced dimensional specification of the system behaviour; a problem that has been studied in the domain of Model Order Reduction (MOR). MOR techniques aim at reducing the order of a dynamical system while preserving the input-output relationship to the extent possible [1]. The robot control applications of MOR techniques is largely underexplored, and we could take inspiration from nature in deriving techniques.

In this context, there is significant biological evidence suggesting that a process of dimensionality reduction may be occurring in neural control mechanisms [7]. The discovery of spinal Convergent Force Fields (CFFs) and their linear combinations in frogs [8] provided neurological justification for the presence of *motor primitives*, which have been described as fundamental units of the motor control system, suitable combinations of which enable complex movements to be carried out. Until now, most approaches have aimed to identify and model primitives from observations of natural movements.

In this paper, we propose a framework for synthesising a motor-primitive inspired control architecture for redundant and compliant robots. The architecture is inspired by recent work in biology [2], which proposed a novel model for the synthesis of motor primitives of a frog's leg using MOR and optimisation of a cost. The work we present adapts and expands their technique to artificial systems, and as a preliminary result we focus on linear dynamical systems. The results are demonstrated in a simulated tendon driven robotic pendulum which uses antagonistically arranged series-elastic actuation.

This paper is organised as follows. The considerations underlying the proposed architecture are presented in Section 2. In Section 3, the algorithm for extracting motor primitives is described. The experiments and simulated results are presented in Section 4, followed by the conclusions in Section 5.

2 Proposed Control Architecture - Considerations

Motor primitives have been characterised [2] as spinally stored constraints on the motor input commands of the form,

$$u = U^{*}C, \tag{1}$$

where, $U^{*} = u^{*}_{1 \cdots k}$ is a set denoted as the motor primitives, comprising of k primitives, and C is the vector of reduced dimensional control inputs, $C =$

$[C_1, \ldots, C_k]^T$. Each column of U^* can be thought of as a set of spinally stored muscle activations, similar to the activations produced by microstimulation of the frog's spine. In this formulation, each primitive u_i^*, is in the dimension of total number of muscles present [2], while only k primitives are needed and used to specify their motion. In this form, the primitives represent the basis vectors of the desired space of motor commands.

The approach of Berniker [2] to motor primitive synthesis used Balance Truncation [1], a control theoretic model reduction approach, to reduce the dimensionality of the mechanical system; the method relies on knowing the mechanical plant model. Furthermore, the approach assumes that motor primitives must be non-negative (real muscles cannot be negatively activated), orthogonal (act independently) and useful for generating commands (formalised based on mathematical properties of the equivalent reduced dimensional system); the primitive computation is based on optimising a corresponding cost function.

Ideally, in autonomous robots the architecture is synthesised in a self-organised manner, without knowledge of the full dimensional model in advance, which requires apriori system identification to be carried out. Also, some of the assumptions underlying the primitive synthesis approach [2] have to be adapted to comply with artificial actuation mechanisms. The technique proposed in this paper makes the following assumptions:

1. **Spontaneous Motor Activity Is Used to Collect a Dataset:** In order to self-organise a reduced dimensional model of the mechanical plant, spontaneous motor activity will be employed to perturb the system and collect a dataset characterising the behaviour, as described in Section 3.1.
2. **Statistical and Data Driven Methods Are Used to Reduce the Dimensionality:** The Oja rule [9] demonstrated the ability of unsupervised learning in a network of neurons, to perform Principal Component Analysis (PCA). Hence for dimensionality reduction, a PCA based method, Proper Orthogonal Decomposition (POD), will be used as described in Section 3.2.
3. **Primitives Can Also Be Negative as Motor Commands Can Be Negative for Artificial Systems:** For robotic systems, inputs may be negative as well, since typically most actuation mechanisms such as DC motors tend to exhibit bi-directionality at the output and bipolarity at the input. We address this consideration by proposing a technique for primitive synthesis using Singular Value Decomposition (SVD) described in Section 3.4.
4. **The Reduced Dimensional Model Is Utilised to Generate Control Inputs to Reach Equilibrium Positions:** Motivated by the equilibrium point hypothesis [7] we propose a reduced dimensional controller that generates required motor commands to reach Cartesian space equilibrium positions as described in Section 3.5.

These new assumptions will be utilised in the synthesis of the control architecture as described subsequently. As a preliminary exploration we shall constraint our proposal to linear dynamical systems, although it can potentially be adapted to nonlinear systems as well.

3 Synthesis Methodology

The proposed reduced dimensional architecture is synthesised using the methodology presented in Fig. 1. The various constituent processes are described in this section.

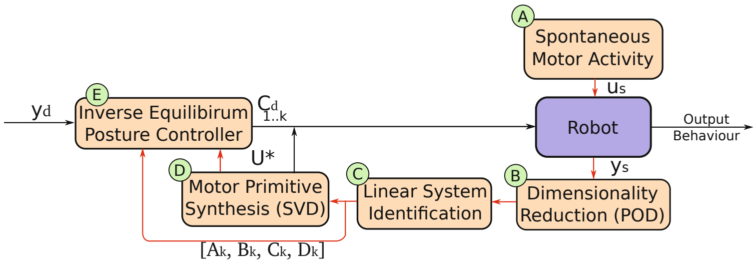


Fig. 1. Motor primitive-inspired Control Architecture - Synthesis Methodology. First the Robot is perturbed by Spontaneous Motor Activity $u(t)$ *(A)* to generate a dataset y_s and subsequently MOR (POD) *(B)* and Linear System Identification *(C)* are applied to yield a reduced order model. Primitives are then synthesised using SVD *(D)* and are combined with the reduced model in the equilibrium posture controller *(E)* to generate motor commands corresponding to desired behaviour goals in end-effector space.

3.1 Dataset Generation through Spontaneous Motor Activity

In mammals, the process of spontaneous motor activity (SMA) carries out muscle contractions in the absence of sensory stimulation. This type of motor activity has been observed during sleep throughout all developmental stages (including the foetal stage) [5]. One particular type of SMA observed is the Myoclonic twitch which spontaneously triggers independent contractions of individual muscles. Inspired by this process, we utilise independent and individual pulse inputs $u_s(t)$ (square signals of amplitude 0.01 and duration 2.8s) to perturb the mechanical system, resulting in a motion output that can be recorded in the form of a dataset (see block *(A)* in Fig.1)of *snapshots* of the dynamical system as $\chi = [x(t_0), \ldots, x(t_i)]$, where $\chi \in \mathbb{R}^{N \times n_t}$ and $x(t_i)$ is the ${n_t}^{th}$ snapshot of the system, where n_t is the total number of snapshots in the dataset (or datapoints) and N is the state dimensionality.

3.2 Reduction Using Proper Orthogonal Decomposition (POD)

The next step is to reduce the dimensionality of the dataset using POD[1] as depicted in block *(B)* in Fig.1. Consider a linear dynamical system of the form below,

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{2}$$

[1] Also called PCA, Karhunen-Loeve decomposition or factor analysis.

where, $u \in \mathbb{R}^I$ is the input, $x \in \mathbb{R}^N$ is the state, $y \in \mathbb{R}^O$ is the output. The matrices A, B, and C are commonly called as the state, input, and output matrices, respectively. In this case, the reduction aims to find a lower dimensional representation z such that,

$$\dot{z} = A_k z + B_k u, \quad y = C_k z + D_k u, \tag{3}$$

where, $z \in \mathbb{R}^k$ is the state, and D is a feedthrough matrix compensating for steady state differences. We thus look to replace the N dimensional system by a nearly-equivalent (similar in behaviour) k dimensional system, where $k \ll N$. From the dataset χ collected in the previous stage, the Singular Value Decomposition (SVD) then can be used to obtain the *best* [11] reduced dimensional approximation $\hat{\chi}_k$ which minimises the norm $\| \chi - \hat{\chi}_k \|_2$. The SVD renders $\chi = U\Sigma V^T$, where $UU^T = I, VV^T = I$, and the singular values are ordered as $\sigma_1 \geq \dots \sigma_k \geq \dots \sigma_n$. We can then truncate χ to the first k singular values, by using the corresponding first U_k singular vectors as a basis of the k-dimensional subspace we are projecting the dataset to, as $z(t) = U_k x(t)$. The next step is to obtain the model parameters.

3.3 Identification on the Reduced Dimensional Dataset

To identify the reduced dimensional model, we employ system identification on the dataset $z(t)$ as depicted in block *(C)* in Fig.1 . Due to the assumption of linear dynamics, the dataset $z(t)$ obtained from POD will also be guaranteed to be linear [1] and linear least squares identification can be employed as,

$$[\dot{z}(t)] = [A_k, B_k][z^T(t), u^T(t)]^T, \quad [y(t)] = [C_k, D_k][x^T(t), u^T(t)]^T, \tag{4}$$

where, z is the new state variable of the dynamical system, A_k, B_k, C_k, and D_k are the reduced dimensional state, input, output and feedthrough matrices respectively. Note that $u(t)$ and $y(t)$ have not changed from the original system in Eq. 2.

3.4 Primitive Synthesis Using SVD

Once the reduced order model is obtained, primitives in the form of Eq. 1 are computed. An important criterion for the primitives is that ideally the commands generated in the reduced dimensional space are "useful" in the sense of their effect on the state [2]. This is ensured by allowing the primitives U^* to be orthogonal to the nullspace of the reduced dimensional input matrix B_k. Moreover, the primitives are orthogonal to each other (to allow spanning the control input space). Both these goals are accomplished by finding the singular vectors of B_k and choosing the last k of these vectors.

Consider the singular value decomposition of a matrix B_k, $B_k = U\Sigma V^*$. The null space of the input matrix B_k has as its basis, the last $n-k$ columns of the right singular vectors V^* of the decomposition [11]. Since the columns of the V^*

matrix are orthogonal to each other, the first k columns thus can be chosen as primitives, since they are both useful and orthogonal.

$$u^* = \mathcal{N}(B_k)^{\perp}, \quad u^* = V^*_{1\ldots k}, \tag{5}$$

where the operator $\mathcal{N}()^{\perp}$ computes the nullspace complement of a matrix. Due to the availability of multiple high speed numerical SVD computing algorithms, primitives computation is faster than methods based numerical optimisation [2].

3.5 Feedforward Equilibrium Posture Controller Design

Once the primitives are computed, a controller can be designed as required. In [4] and [8], it is suggested that the controller is feedforward in structure and it generates the necessary commands to affect the equilibrium posture of the limb. For the obtained system in Eq. 3, the equilibrium state for a given input corresponds to $\dot{z} = 0$, which is therefore,

$$z = -A_k^{-1} B_k u, \quad y = \left[-A_k^{-1} B_k + D_k\right] u. \tag{6}$$

Since the input u is constrained according the motor primitive as in Eq.1, it is sufficient to compute the required reduced dimensional control inputs C_d for a desired output y_d where $C_i \in \mathbb{R}^k$. For this, the pseudo-inverse or the Moore-Penrose inverse (†) can be used to obtain,

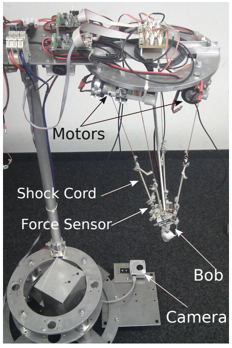
$$C_d = \left[\left(-C_k A_k^{-1} B_k + D_k\right) u^*\right]^{\dagger} y_d. \tag{7}$$

Note that if k is chosen to be of same dimensionality of the output y, Eq.7 is computed using a regular inverse instead of the pseudo-inverse and thus the redundancy problem is directly resolved.

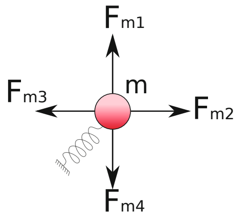
4 Experiments and Results

4.1 Methods: Pendulum Robot and Simulation

The pendulum robot platform is a test setup built to investigate methods and techniques for developmental robotics. Loosely inspired by the human shoulder system, it consists of two mechanically independent pendula, each driven by 4 series elastic actuators coupled in an agonist-antagonist configuration, as shown in Fig.2a. Each muscle system can be actuated independently and includes force and elongation sensors. A camera is mounted on the base of the pendulum looking upwards to extract end-point position as a 2D position measured in the camera frame of reference. The dynamics of this robot is be assumed to be linear under the conditions of bounded amplitude motion due to the relatively long length of the muscles. Since the platform is driven by 4 motors, the input dimensionality is 4. Thus in order to be interesting, we must synthesise a controller with k primitives where $1 < k \leq 4$ to perform meaningful tasks in the 2D task space.



(a)



(b)

Fig. 2. a) Pendulum robot platform and b) Linear System simulation of the Pendulum robot platform (the mass is that of the end-point bob)

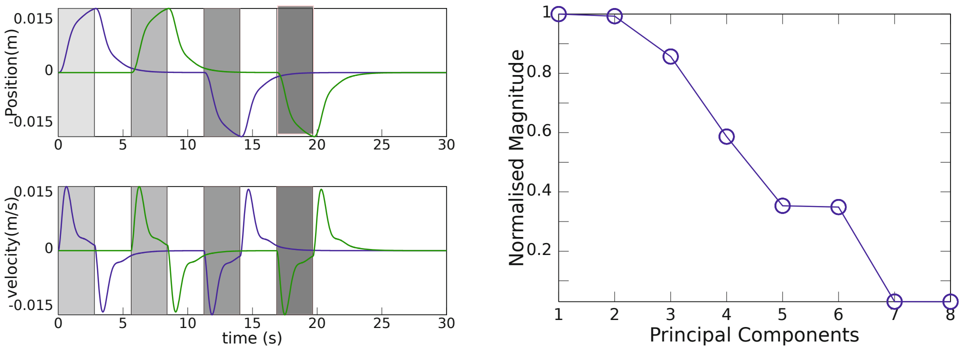
The simulator depicted in Fig.2b uses a linear approximation of the plant. The nonlinearities due to angles of force application are neglected as a simplification. The simulation implements the following model,

$$\begin{aligned} \ddot{x}_c &= -k_x x_c - b_x \dot{x}_c + \sum_{i=1}^{4} F_{m_i} cos(\theta_i), \\ \ddot{y}_c &= -k_y y_c - b_y \dot{y}_c + \sum_{i=1}^{4} F_{m_i} sin(\theta_i), \\ \dot{\alpha}_i &= \tau(u_i g - \alpha_i), \quad F_{m_i} = k_m \alpha_i, \end{aligned} \tag{8}$$

where, $k_{x,y}$ is the stiffness of restoring force to mean position, $b_{x,y}$, is the damping of the pendulum bob, $i \in [1, 4]$ $\tau_{1 \cdots 4}$ are the time constants of the muscle (critically damped), $g_{1 \cdots 4}$ are the gains on the input signal, k_m is the muscle stiffness proportionality to its activation, and θ_i is the mounting angle of each of the spindle motors. Note that, x_c and y_c in this model are both state variables and should not be confused with the state x and output y of Eq.2. For this paper, the simulation constants were fixed as $k_{x,y} = 10$, $b_{x,y} = 5$, $\tau = 1$, $g = 5$, and $k_m = 5$ (currently being validated on the real robot platform). The model has dimensions 8 on state and 4 on input, corresponding to desired angular positions on DC motors with controllers of time constants τ and dc gain $g_{1 \cdots 4}$. The model was implemented in GNU Octave and integrated using the *ODE45* routine.

4.2 Results - Spontaneous Motor Activity and Dimensionality Reduction

A unit pulse input applied to each muscle sequentially to replicate the spontaneous motor activity in the form of single muscle twitches as shown in Fig.3a. The various state and output trajectories $y_s(t)$ and $x_s(t)$ respectively, were recorded and stored as a dataset and POD was used to reduce the dimensionality.



(a) (b)

Fig. 3. a) Single Muscle Twitching and output of the simulated robot and b) Principal components of the dataset $(y_s(t))$ -Scree Plot

The principal components are depicted in the scree plot in Fig.3b. The first k largest components were chosen to compute controllers with k motor primitives, where $1 < k \leq 4$, thus giving 3 types of controllers. Linear dynamical models of dimension k were then obtained by fitting in each case.

4.3 Results - Synthesised Motor Primitives and Task Performance

From the identified reduced dimensional linear state space, the primitives were synthesised using SVD for the cases of $k = 2, 3$, and 4 primitives. The computed primitives in each case are depicted in Table 1. The synthesised primitives are visualised by locating the resulting equilibrium points (at $\dot{x} = 0$) for a unit inputs applied to each of the reduced dimensional input C individually. Since the source system is linear, the equilibrium points obtained are unique and independent of the initial conditions as depicted in Fig. 4. Since any position in the Cartesian space can be obtained by using the right input C, the knowledge of these equilibrium points can be used to generalise to new points in the task space by using linear combinations similar to the biological case [8].

A reaching task was then computed using the controller form of Eq. 7 for a set of 3 controllers ($k = 2, 3, 4$), as shown in Fig. 5. In each case small offset errors result in steady state due to the quality of the obtained reduced dimensional model. This offset error could potentially be minimised if the model can be improved through subsequent stages of learning and adaptation. More complex

Table 1. Computed U^* for the cases of 2, 3, and 4 primitives

$k = 2$		$k = 3$			$k = 4$			
-0.53210	-0.49997	-0.56734	-0.41926	0.49501	-0.55065	-0.44407	-0.50324	0.49632
0.46796	-0.49313	0.56418	-0.56742	-0.34017	0.57413	-0.53848	0.34869	0.50874
0.54176	0.41663	0.43817	0.57504	0.48379	0.40476	0.59823	-0.47541	0.50227
-0.45208	0.57730	-0.40967	0.41423	-0.63655	-0.45091	0.39365	0.63178	0.49252

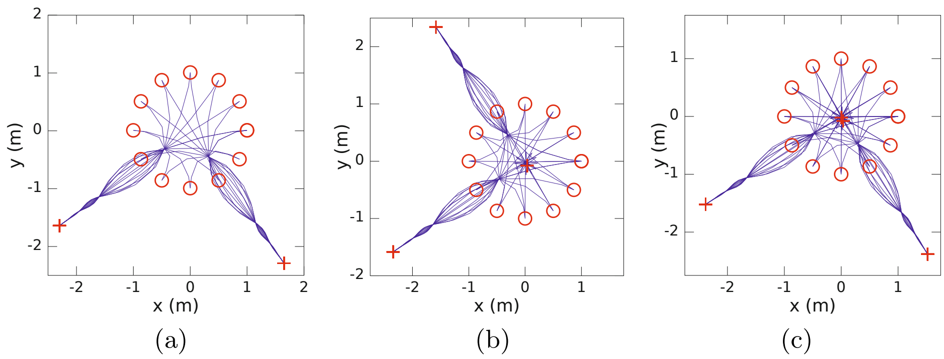


Fig. 4. Equilibrium positions of endpoint (*red* stars) while using (a) $k = 2$, and individual unit inputs in C i.e. $C_1 = 1, C_2 = 0$, and vice versa (b) $k = 3$, and individual unit inputs in C as before, and (c) $k = 4$, and individual unit inputs in C as before, the initial conditions (*red* circles) are chosen to lie in a circle about the center. The trajectories of the endpoints are in *blue* lines. In cases (b) and (c) the latter equilibrium points are are found to lie nearly at the origin of the workspace.

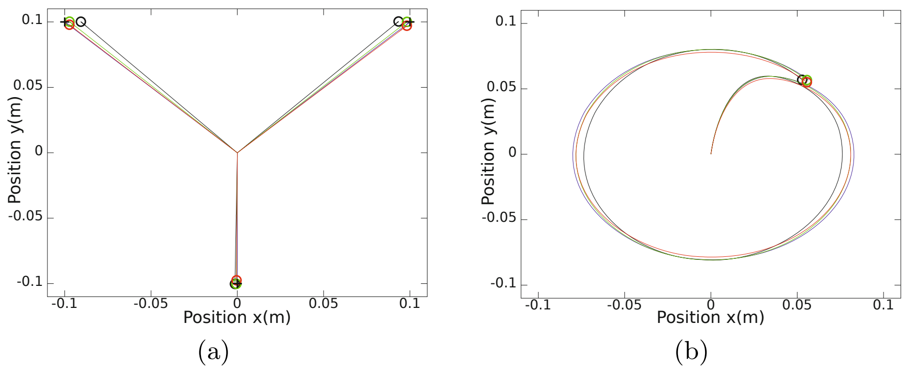


Fig. 5. Performance of the 3 controllers, $k = 2$ (*red*), $k = 3$ (*green*) and, $k = 4$(*black*) relative to an ideal controller (*blue*), in performing (a) reaching task in Cartesian Space to the positions $(0.1, 0.1)$, $(0, -0.1)$, and $(-0.1, -0.1)$, (b) continuous tracking task in Cartesian Space to a circle centered at $(0, 0)$ and diameter 0.15m. The trajectories obtained in each case are nearly identical.

desired trajectories using multiple waypoints can also be obtained using the controller as shown in Fig. 5.

5 Conclusions

This paper presented a motor primitive inspired architecture for reduced dimensional control of redundant compliant robots. Based on a biological model of motor primitives using model order reduction, considerations relevant to artificial system control were presented. A technique for self-organising a controller was presented, inspired by the concept of spontaneous motor activity. A reduced dimensional representation of the ensuing dataset was then used to synthesise motor primitives using SVD. The computed primitives were then utilised to compute the necessary control, across all of the inputs, for reaching fixed points in space. The proposed framework was tested on simulated version of a compliant redundant tendon driven robot platform. The preliminary simulation based results are promising and demonstrate the utility of the proposed technique for application to artificial systems. From an engineering viewpoint an extension of the work to the nonlinear systems such as kinematic chains is currently being carried out. An important consequence for biological systems arising as an extension of this work is an investigation on the relationship between dimensionality reduction and mechanical properties of biological systems.

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Appendix C

Developing Learnability - the Case for Reduced Dimensionality

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Developing Learnability – the Case for Reduced Dimensionality

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***Abstract*—In this work, the notion of reduced dimensionality and its relevance for systems undergoing development is examined. The various motor control theories of degree of freedom change, optimal control, and motor primitives are related using the framework of control dimensionality reduction. Based on their relationship, we propose a developmental approach based on progressively utilising increasingly higher dimension representations of the system. A simulated planar 2 link arm model is then used to demonstrate the effect of utilising reduced dimensional models for control; comparisons on step and sinusoidal tasks are presented showing a progressive decrease in error that is task dependent quantitatively. Arguments are presented for why such a strategy might be essential from an evolutionary perspective for the developmental acquisition motor control in a tractable manner.**

I. Introduction

Biological systems have two fundamental and conflicting properties: high dimensionality and learnability. Both are products of evolution. Learnability is the ability of an individual organism to self-modify and 'learn' behaviours in different environments during its lifetime. It is a form of phenotypic plasticity that allows individuals phenotypes to fine-tune their performance according to their local 'environments'. Most organisms undergo a prolonged period of irreversible changes that involves physical growth (maturation) and an increasing repertoire of complex behaviours that are dependent on experience (developmental plasticity). Fitness, in this context, is finding developmental trajectories that reach high adult fitness [12]. But even adult organisms are constantly adapting to changing local environments (e.g. adaptive control, motor and cognitive learning). During any learning phase, the organism is sub-optimal in its behaviour, and it would pay to reach the optimal behaviour as quickly as possible, in a developmental sense. Thus, learnability should be a phenotypic trait that also needs to be optimised.

The problem of high-dimensionality was crystallised by Bernstein's [3] 'degrees-of-freedom (DOF) problem' on how to control systems with large numbers of structural DOF (joints etc.) in a stable manner. The problem extends to control dimensions (or order) of a dynamical system (attractor dimension) where a large number of variables need to be controlled in time [19], [22]. Regardless of the state space in which the analysis is performed, invariant behaviours have been observed in human motion indicating some form of decreased dimensionality; eg.piecewise behaviours (e.g. arm reaching) tend to have rather low dimensionality, implying redundancy (per task). The question of how this dimensionality reduction is accomplished is a major theme in motor neuroscience with huge implications for robotics as well.

The general solutions proposed in resolving this problem can be grouped under three broad classes : (1) Active dimension control, i.e. freezing and freezing of DoF, (2) Optimal Control Theory (OCT) approaches,(3) Motor primitives and modular control architectures; each of these proposals tackle one aspect of the dimensionality issue.

The first approach of active control of dimensionality was by Bernstein himself who proposed a three stage learning process involving freezing of distal DoF during task learning followed by progressive unfreezing, i.e. a gradual dimension increase. Many researchers have sought to find dimension increase in human skill learning although evidence exists in child motor skill development [4], [5]. An alternative viewpoint from the dynamical systems perspective [19] is that in task learning, we instead decrease the active DoF of an attractor. This notion has also been explored in developing robots, where initial stages of exploration are forcibly constrained with a freezing of distal kinematic DoF [18]. Although the general focus of these approaches is on how controllable DoF are brought into play in task completion, none of these proposals address how the coordination resolves the redundancy present in the full state space of the system, neuro-mechanical or otherwise.

This preference has instead been tackled by optimal control theory (OCT), where behaviours are chosen because they minimise some Lagrangian. Numerous Lagrangians have been proposed; those involving unweighted or weighted Euclidean norms of control input have been particularly successful at capturing observed biological behaviour [10], [13], [23]. The OCT approach allows computation of control for a desired task by careful choice of the Lagrangian; the appropriate behaviour can thus be identified from a set of possible solutions. However, the OCT approach does not address how a low dimensional preferred behaviour can be learnt within a high dimensional manifold. Exploration time, local optima, and inverse non-uniqueness are some of the curses of dimensionality, and would be expected to adversely affect developmental fitness.

The third viewpoint is that of simply constraining the input into some linear combination of modular system; this leads to movements being composed of a vocabulary of "primitives". Thus the aim is to reduce the overall dimensionality of phenotypic space, particularly by modularity where different subsets of dimensions can be optimised in parallel and then combined hierarchically into more complex spaces [12]. For motor systems, this would be achieved by reducing control input dimensionality with low-dimensional 'motor primitives' as a basis set for reduced model control [7]. The biological notion of motor primitives [21] (stored motor patterns or muscle synergies [9]) is a key concept in the motor neuroscience literature [17], and provides a model of modular organisation of motor control.

Here we relate some of these notions by two key ideas. First we show that motor primitives which are synthesised by reduced dimensional techniques also lead to norm minimisation at the input; dimensionality reduction through projecting the state into a subspace with minimised norm is thus proposed as a viable technique. In the second part of the paper, we then directly apply dimensionality reduction on the dynamics using a technique of empirical Balancing Truncation. We then utilise progressively higher dimensions on two kinds of benchmark tasks to show how progressive dimension change leads to qualitative improvements in behaviour for a nonlinear arm system.

This paper is organised as follows. Section II presents motor primitives and minimum norm control as reduced dimensional control strategies followed by a brief review of balanced dimensionality reduction. Section III discusses the arm model we employ. Section IV presents the simulation results on dimensionality reduction and control on the arm model. This is followed by a discussion on the notion of learnability and reduced dimensionality in Section IV.

II. Minimum Norm Control, Motor Primitives and Dimensionality

First, we briefly review the approaches of minimum norm control and motor primitives and relate them using the notion of dimensionality reduction. Consider the dynamics of the musculo skeletal system represented by,

$$\dot{x} = f(x) + Bu, \quad y = h(x) \tag{1}$$

where the function $f(x)$ represents the state transition dynamics (natural dynamics) and B is the input matrix. The output y relates to the full order state x through the function $h(.)$. Note that $x \in \mathcal{R}^N$, $y \in \mathcal{R}^O$, and $u \in \mathcal{R}^I$, where N is the dimensionality of the state, O the dimensionality of the output and I that of the input. The aim of control is to synthesise an input $u(t)$ for some desired task $y_d(t)$.

The Minimum-*Effort* control was proposed by Daunicht [8] and is related to Pseudo Inverse Control (PIC) in robotics literature [14]. Subsequent works in the eye movement literature [10] demonstrate that human oculomotor data seem to follow this principle. In particular, it is effective in explaining the motor commands for human post-saccadic fixation, the behaviour of maintaining a fixed eye position after a saccadic motion. Minimum norm is relevant since other techniques for OCT also utilise Lagrangians that are based on norms on input [13]. For the system in Eq.1, minimum norm control is computed by minimising the cost function,

$$J_{Min-Norm} = \|u\|_2, \tag{2}$$

for the desired task $y_d(t)$. Although simple in formulation, for even moderately high dimensional spaces, computing an optimal control might be intractable.

An alternate kind of constraint on the input that discussed in motor control literature is that of motor primitives, proposed as a key modular organisational principle underlying motor control in biology. Motor primitives are sometimes defined as a constraint of the input into the linear summation of the form,

$$u = W^* \tilde{u}, \tag{3}$$

where $\tilde{u}$ is considered to be a new set of inputs combining the primitives defined in this case by W^* [7], [21]. The primitves may also be considered to be patterns in time $W(t)$, also known as muscle synergies [9]; existing models addresses two questions, (1) number of primitives (2) criteria for their selection.

In this context, it has been shown that primitives might be synthesised by knowledge of a reduced dimensional representation of the dynamics of the musculoskeletal system [2]; the dimensionality of the reduced system provides the number of primitives. The other choice of primitives is dictated by them acting as a basis set for specifying the control inputs; ideally they span the full dimensional space of input commands to the extent possible with minimal overlap [16]. The usage of primitives essentially leads to the *projection* of the dynamic behaviour to a lower dimensional manifold as in Eq. 4.

From a control perspective, reduced dimensionality techniques aim to compute the control input through utilisaton of the dynamics of the form,

$$\dot{z} = f_r(z) + B_r u, \quad y = h_r(z), \tag{4}$$

where z is a reduced dimensional state variable which is given by $z = \mathcal{P}x$ relating the input u to the output y though the new reduced system matrices of $[A_r, B_r, C_r]$ where $\mathcal{P}$ is called the *Projection matrix*. Note that while ideally we look for a model that produces an identical relationship between u and y, in practice, an approximation of the output $\hat{y}$ is obtained. The well known techniques [1] for projection attempt to minimize the norm of projection,

$$J_{Projection} = \|z - \mathcal{P}x\|_2. \tag{5}$$

It must be noted that a key consequence of the criteria for choice of primitives W^* is that the resulting product $B_r W^*$ is maximised, i.e. "*Best excite the natural dynamics*" [2]. Since this maximum is obtained, it can therefore be concluded that the resulting input signal thus computed, $W^* \tilde{u}$ for any desired trajectory y_d also minimises the norm, $\|u\|_2$ since their product is constant and related to the desired reduced dimensional state z_d.

The problem of control learning can therefore be simply be reduced to that of acquiring a dynamic representation of the system behaviour that minimises the projection norm of Eq.5. A control computed on this system would also be equivalent to the minimum norm solution of Eq.2.

We thus hypothesise that reduced dimensionality might be an organisational principle underlying development. We now discuss how reduced dimensionality is related to the dynamics and how a reduced dimensional representations of increasing dimensionality may be computed for a nonlinear system such as a jointed limb.

Dimensionality Reduction though Balancing

Data dimensionality reduction is a well understood technique used in a wide variety of contexts.

However, the notion of model dimensionality reduction is a relatively recent engineering approach employed in the simulation and control of complex systems [1]. While techniques such as PCA (also known as Proper Orthogonal Decomposition or POD) deal with reduction in dimensionality of state based on its statistical properties, when the control of systems is to be computed, it is more important to consider input-output stability - the principle underlying Balanced Reduction [20].

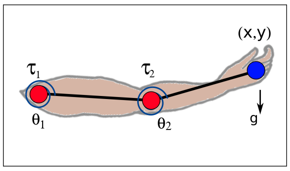
We first seek a rotation of the state of a system known as Balancing Transform $\mathcal{T}$. This is done on the basis of Hankel Singular Values (HSV) Σ computed as the square root of the product of the controllability Gramian Matrix ($\mathcal{P}$)- measuring relevance of states to the input and the Observability Gramian Matrix$\mathcal{Q}$ - measuring relevance of state to the output. The HSVs are a score by which states can be eliminated on the basis of their importance for input and output. The controllability and observability gramian matrices have closed form solutions for linear systems, and approximate solutions for some classes of nonlinear systems [11]. Once we obtain $\mathcal{T}$, the Projection matrix for dimensionality reduction can be computed as,

$$\mathcal{P} = [I_r, 0]\mathcal{T}, \tag{6}$$

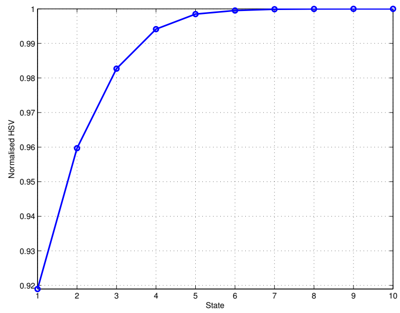
where I_r is a identity matrix of dimensionality $\mathcal{R}^{r\times r}$ for a system reduced to dimensionality r. Using this projection matrix with variour values of r allows the computation of the reduced dimensional models in Eq.4.

III. Arm Model

We utilise a simplified planar arm model of Fig.1a for our experiments. It consists of 2 joints with the angles θ_1, θ_2 along with joint compliance (series elastic) actuated by 2 muscles each producing the torques τ_1 and τ_2 at the two joints respectively. For simplicity, we assume that the muscles produce torques proportoinal to activatons $[\alpha_1, alpha_2]^T$ that are first order delayed response to neural input $[u_1, u_2]^T$. The system has state dimensionality of $N = 10$ represented by, $x = [q_1, q_2, \dot{q}_1, \dot{q}_2, \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \alpha_1, \alpha_2]^T$, where $q_{1,2}$ are the joint angles of the arms and $\theta_{1,2}$ are the state of the series elastic compliance.



(a)



(b)

Fig. 1. (a) Model of the arm used for the experiments, (b) Normalised Hankel Singular Values of the system.

The dynamics are represented in the form of Eq.1, with the joint angle ouputs $y = [q_1, q_2]^T$. The arm has frictional damping at the joints, and is acutated under gravity. The model was simulated on *Matlab 2012a* and the equations were integrated using the *ode15s* routine of the *odepkg*. The matlab routines of the model and the simulation source are available on request.

IV. Results

We synthesise a reduced dimensional representation of the arm dynamics using the nonlinear balancing technique. First, impulse-like signals are applied to the inputs of the arm and resulting behaviour is utilised to compute $\mathcal{T}$ and the normalised HSVs σ_i as shown in Fig. 1b.We can then compute a suitable Projection $\mathcal{P}$, from Eq.6. Since the HSVs indicate that all models of dimensionality greater than 3 will be suitable, we synthesised reduced dimensional models of progressively increasing dimensionality from 3 until 10.

The quality of each of these reduced dimensional models is then compared against the full

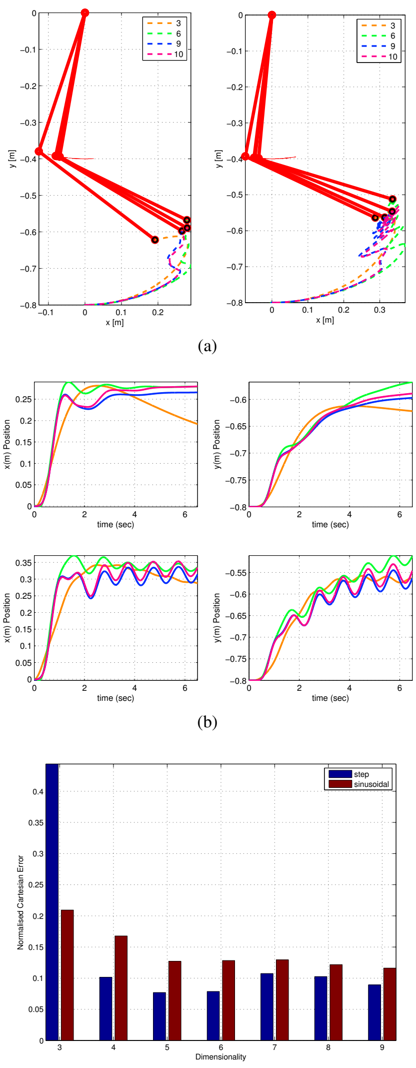


Fig. 2. Comparison of trajectories of full and reduced order models; (a) Step Response, (b) Sinusoidal Response; Full and reduced order trajectories (dimensionality 3, 6, and 9) are dashed lines of varying colours. (c) Error in step and sinusoidal responses of the reduced dimensional system decreases with increase in dimensionality.

dimensional system behaviour. Fig. 2 shows the comparisons for a step and sinusoidal responses. The close correspondence of the reduced dimensional systems of increasing order with the full dimensional system can be seen in both the cases.

The cartesian end position in Fig.2a shows how each of the trajectory can compare in the elicited behaviour; thus the reduced dimensional models can be used for planning and control of various tasks. Furthermore, from Fig.2c, it can be seen that the cartesian errors decrease with increasing dimensionality, although the decrease in error is dependent on the task. Thus it might be possible to choose the smallest possible dimensionality in order to learn a task, and then progressively use higher and higher dimensions to learn more complex tasks.

A key result is to quantify the progressive decrease in control error with usage of increasing dimensionality. The result presented in Fig.2c measures the norm of cartesian position error throughout the whole trajectory between the reduced and full dimensional models. As it can be seen, usage of increasing dimensions consistently decreases the error. This indicates that if a developmental strategy is related to dimensionality reduction, computation of globally optimal solutions may be carried out with the usage of increasing dimensionality in the internal representations and optimisation of motor control. A natural consequence of computing solutions with the lower dimensional models is tat learning optimal trajectries in the lower dimensional space is faster, thus redering the entire development control learning process tractable.

An important consequence of this development is the ability to progressively employ the increased number of degrees of freedom on more complex tasks. We quantify this notion by comparing the step responses and a sinusoidal responses in Fig.2a,b. The error change qualitatively similar for both tasks, although different quantitatively; thus increasing task complexity may be related to employment of increasing dimensions in the control, without compromising stable control of tasks learned earlier.

V. Discussion

Mininum norm control (pseudo-inverse control) is often considered in robotics as a means of achieving control of a redundant system in the least squares sense, i.e. generating inverse dynamics. It essentially solves the Bernstein's DOF problem (although modifications to reduce sensitivity to singularities may be employed). It has also been proposed that the real (biological) control of eye movement statics (fixation) and hand movements is also under MNC control [10], and it has been implicitly assumed that this naturally occurring MNC is a biological example of redundancy control. However, we have shown this is equivalent to reducing dimensionality through projection. Furthermore, in our example, we demonstrate how increasing dimensions in the motor control can lead to progressively decrease in control errors. Thus, reduced dimensionality entails finding a low-dimensional manifold that allows simplified control to a satisfactory accuracy.

Dimensionality and Meta-Optimization

This insight provides a significant alternative view to understanding why MNC seems to fit biological data. Instead of appealing to OCT, we propose instead that MNC reflects the process of dimensional reduction in nature. We consider the projection to be a phenotype parameter (trait) of an individual that may change with age [22] (whether due to experience, genetic control, or both). In general, different primitives will have differing costs and benefits for the organism resulting in different evolutionary fitnesses. Reducing dimensionality allows for faster learning and optimization (in reduced dimension). At a meta-level there will be a speed-accuracy learning trade-off. Learning very precise control requires a long time, but fast learning is inaccurate. For our simple model, this is shown in Figure 2c.

Thus, depending on the organism's context it may pay to change. For a young naive infant (robot) the time taken to learn a task with a full system may be prohibitive, so reduced dimensionality would be preferable. But as the infant's learning progresses, it may pay to increase dimensionality to improve accuracy, as has been proposed, (cf. 'unfreezing' DOF [22]). Indeed, there may be an optimal policy for changing to reach a desired accuracy in minimal time.

When confronted with different tasks (i.e. contexts), different projections may be useful. Thus, the same system can be utilised for different goals. For example, we expect steady-state and dynamic projections to be different. Thus, switching between different projections would allow different tasks to be learnt (at different rates). This is a similar to the idea of switching between context-dependent internal models [24], but in the domain of development. This notion can also be related to the kinematic notion of alternately freezing and unfreezing degrees of freedom, that is well known in developmental robotics literature [6].

Another key use of dimensional reduction lies in hierarchical learning, where low dimensional primitives are learnt first, and then combined to learn more complex patterns of behaviour. This speeds up overall learning but requires the overseeing of the timing of different phases of learning – a 'developmental trajectory' [12]. We have not explored this phenomenon here, but it requires the ability to modulate learning-rate. In natural development this is observed as a phasing in and phasing out of sensitive periods, which are controlled by a complex feedback system of genes, neurotransmitters, and growth factors operating pre- and post-natally. In our view, dimensional reduction in artificially learning agents necessarily requires such master control of learning rates, which presumably is learnt itself ultimately on an evolutionary time-scale. Modulation of the learning rate can be accomplished as a dimensional change in a form similar to Bernstein's original suggestion. Thus, dimensional change may be closely coupled with the morphology and material property change during development [15].

An important but unanswered question is the effect of the underlying motor structure on dimensional reduction. In general, the optimum in reduced dimensions does not coincide with the global optimum of the full system. Thus, speeding up learning may drive the system to a suboptimal state. The choice of primitives appears to be more important than dimensional reduction, *per se* [2]. In any case, for biological organisms, this may be advantageous over an extended period of development. This is a complex problem that depends not only on the choice of primitives, but also on motor anatomy. It seems highly unlikely that motor anatomy has evolved independently of motor learnability, and we expect both to co-evolve as two phenotypic traits. Indeed, Daunicht [8] offers the insight that the judicious placement of joint sensors allows an optimum (and deviations therefrom) to be rapidly found. Thus, the design of learnable motor systems, such as a developing robot, needs to consider dimensional reduction as a core parameter.

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Appendix D

Muscle Synergies and Dimensionality Reduction

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Do muscle synergies reduce the dimensionality of behaviour?

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Research Topic

ABSTRACT

The muscle synergy hypothesis is an archetype of the notion of Dimensionality Reduction (DR) occurring in the central nervous system due to modular organisation. In order to validate this hypothesis, it is important to understand if muscle synergies can indeed facilitate accurate real-time control and optimisation of motor behaviour. In this paper, we investigate this problem by synthetically examining the reduction of spatio-temporal behaviour dimensionality due to control using muscle synergies. Our approach is based on the observation that control in the form of temporal muscle synergies constrain the dynamic behaviour of a system in trajectory-specific manner due to the synergy weight matrix. We then use system balancing to define a normalised Hankel Singular Value (HSV) measure for quantifying the DR of this constrained system; we term this approach as Trajectory Specific Dimensionality Analsyis (TSDA). We then develop a model for Minimum Dimensional Control (MDC) to find the optimal weight matrix corresponding to the minimum dimensional trajectory that satisfies all of the task constraints. The TSDA and MDC methods are tested on simulation on linear (tethered mass) and nonlinear (compliant kinematic chain) system; dimensionality of various reaching trajectories is compared and idealised synergies of Legendre polynomials, and Fourier bases are tested. We show that smooth straight-line Cartesian trajectories with bell-shaped velocity profiles emerge as a minimum dimensional solution to reaching tasks in linear and nonlinear systems. The results indicate that a system, synergy profile and trajectory-specific DR of motor behaviours results from usage of muscle synergy control. The implications of these results for the synergy hypothesis, optimal motor control, and developmental skill acquisition are then discussed.

Keywords: Modular Motor Control, Muscle Synergies, Dimensionality Reduction, Hankel Singular Values, Optimal motor control

1 INTRODUCTION

There is increasingly a consensus that the solution to the Degree of Freedom (DoF) Problem of Bernstein (1967) involves some form of dimension reduction (DR) due to modularisation, although it is unclear how this occurs. Aside from uncovering physiological evidence for modular structures in the CNS, a key aspect of the problem is in understanding whether DR can really facilitate natural motor behaviours. While DR can expedite learning and real-time control in some cases, it can also constrict the functionality of the neural mechanisms. It is therefore important to examine the functionality of a set of modules in achieving effective motor control in an organism. Of the many kinds of modules that have been proposed (Flash and Hochner, 2005), the muscle synergy hypothesis, typified by coordinated activation of groups of muscles, has in recent times gained significant attention (Alessandro et al., 2013).

Spatio-temporal regularities in activation patterns across many muscles that seemingly are task and subject independent is usually cited as evidence for DR in the muscle synergy hypothesis (Tresch et al., 2006; d'Avella et al., 2003; Hart and Giszter, 2004; Ivanenko et al., 2004; Ting and Macpherson, 2005). Although various formulations have been proposed, there are some common features: (i) there is a task-specific recruitment of task-independent modules, (ii) the synergies themselves are considered as input-space generators (d'Avella et al., 2003), (iii) the number of modules available for recruitment represents a DR of the control input (Ting, 2007; Chiovetto et al., 2013), and (iv) there is a linearisation of the highly nonlinear musculo-skeletal control problem (Berniker, 2005). It has been surmised that each of these features facilitates real-time control and speeds up motor learning. Nevertheless, a recurring criticism of the hypothesis is its phenomenological nature and difficulty of falsification (Tresch and Jarc, 2009; Kutch and Valero-Cuevas, 2012). One approach towards validation is to synthetically examine the functionality of synergies in facilitating control (Neptune et al., 2009; McKay and Ting, 2012; de Rugy et al., 2013) – i.e the task-space perspective (Alessandro et al., 2013). Along these lines, our approach is to examine synthetically if synergies can facilitate optimal motor skill development.

Berniker et al. (2009), proposed a synthesis technique for time-invariant synergies that was based on model and control dimensionality reduction. A task-relevant reduced dimensional dynamic model was used for both synergy synthesis, and planning; near-optimal trajectories emerge with a set of just a few synergies. Although this method obtains synergies that closely correspond with those extracted experimentally, it must be noted that this time-invariant synergy formulation does not conveniently encode the temporal complexity of natural behaviours. For instance, in the analysis of locomotor movements it has been shown that temporal synergies (Ivanenko et al., 2004, 2005) are more effective in capturing the temporal aspects at various instances within a gait cycle. In this formulation, the synergies can be interpreted as a pool of task-independent fixed temporal patterns that are selectively recruited in a task-dependent manner for generating the necessary muscle activation (Chiovetto et al., 2013). This formulation has also been used to model motor skill development; an increasing pool of synergies is seemingly employed by adults when compared with infants (Dominici et al., 2011), or in allowing increased behavioural complexity (Ivanenko et al., 2005). Temporal synergies are therefore ideal candidates for exploration of the role of synergies in learning and development.

While the task-space perspective has been taken into consideration in the extraction of the so-called functional muscle synergies (Ting and Macpherson, 2005), care must be taken in identifying the objectives of motor behaviour. For instance, kinematically invariant trajectory features such as the bell-shaped velocity profiles in point-to-point reaching (Hogan, 1984a) or the two-third power law in handwriting (Viviani and Flash, 1995) suggest that optimality principles might underlie the motor control. Various hypotheses have been proposed for the performance index that seemingly is reduced (Todorov, 2004; Harris, 1998a); this is also dependent task context (Harris and Wolpert, 1998). However optimal control hypotheses usually do not address how such a control might be acquired by an organism. Phenotypic variability implies that this optimal motor control cannot be innately specified but must be acquired (Sporns and Edelman, 1993); this usually results in life-long adaptation of control. High dimensionality of state, as well as input, can lead to nonviable learning/developmental tasks for real organisms with finite time horizons, that is, the '*learnability*' of optimal behaviour is affected (Kuppuswamy and Harris, 2013). It has been suggested that a developmental strategy of exploration of progressively increasing sensorimotor space dimensions enables this "*curse of dimensionality*" to be circumvented (Vereijken et al., 1992; Sporns and Edelman, 1993; Ivanchenko and Jacobs, 2003), although it is entirely clear how this is takes place. In this context, muscle synergies should not only be considered as input-space generators, but must also be viewed as facilitators of motor skill development. In order to test this supposition, we re-examine the role played by muscle synergies in dimensionality reduction.

In this paper, we address these issues in the following way. We first develop a synthetic quantification of the reduction in spatio-temporal dimensionality of motor behaviours due to control using synergies – this enables assessment of their role as facilitators of learning and control of motor abilities. The temporal synergy formulation we utilise is composed of a task-independent pool of orthonormal basis patterns – that the dynamic behaviour of the system is uniquely specified by the weight combinators which are computed in a task-dependent manner. Our approach is based on a constrained reformulation of the dynamics of a given system wherein instead of motor commands, the synergy patterns are effectively treated as control inputs that are simply turned '*on*' for the duration of a movement. This is based on the observation that temporal synergies are characterised by a dominant timing sequence that are seemingly independent of sensory feedback (Ivanenko et al., 2005). We then reduce the dimensionality of this reformulated system using a method of system balancing (Moore, 1981; Lall and Marsden, 2002; Hahn and Edgar, 2002) – the best subspace that captures the input (synergy basis) and output (task space) is computed using a threshold on a Hankel Singular Values (HSVs) measure. This approach, that we term as Trajectory Specific Dimensionality Analysis (TSDA) obtains both the dimensionality and the subspace dynamics of the system following a given trajectory. We test this approach on both linear (tethered mass) and nonlinear dynamical systems (compliant kinematic chain) ; the dimensionality of various reaching trajectories is compared when using synergies composed of Legendre polynomials and Fourier bases. The results show how synergies can accomplish reduction of behaviour dimensionality that is trajectory dependent and basis-specific.

We then utilise the TSDA to identify the minimum dimensional trajectory satisfying task constraints for a given dynamical system utilising a given basis set of temporal synergies. We develop a cost function for dimensionality utilising the HSV measure. Numerical optimisation is used to compute the synergy weights that correspond to a minimum dimensionality cost while satisfying the task constraints. Using our model Minimum Dimensional Control (MDC), we show that smooth trajectories with bell shaped velocity profiles are the minimum dimensional trajectory for reaching tasks; the velocity profile depend on the temporal characteristics of the synergy basis employed. In the nonlinear case of the kinematic chain, near straight line Cartesian trajectories are obtained as the minimum dimensional solution. From the simulation results, we observe a close similarity of the trajectories to human motions, thus indicating that minimum dimensional principles might underlie motor control using synergies.

We introduce our approaches in the following way: In section 2 we first outline the temporal synergy control problem and review dimensionality reduction and the method of system balancing. Subsequently we derive the TSDA and our proposed minimisation model, MDC and the simulation experiments are described in section 2.5. The simulation results on the linear tethered mass system and the nonlinear compliant kinematic chain using Fourier and Legendre polynomial basis synergies are presented in section 3. We then discuss the implications of our results in section 4.

2 MATERIAL & METHODS

We first introduce some basic formalism to the temporal synergy control problem. Consider the following representation of the neuro-mechanical dynamics,

$$\mathbf{y}(t) = h(\mathbf{x}, t), \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, \mathbf{u}, t), \tag{1}$$

where the variables $\mathbf{x}(t)$ denotes the state, $\mathbf{u}(t)$ the input, and $\mathbf{y}(t)$ the output. For this system the state-space dimensionality can be described by $\mathbf{x}(t) \in \mathbb{R}^N$, the inputs lie in $\mathbf{u}(t) \in \mathbb{R}^{N_i}$, outputs $\mathbf{y}(t) \in \mathbb{R}^{N_o}$ and N_i and N_o need not be equal to N. We utilise a continuous control system description, so $\mathbf{u}$ can be considered to lie in the infinite dimensional space of continuous functions. Let us define this system by $\mathcal{F}(f(\cdot), g(\cdot), h(\cdot))$, where, $\mathcal{F} \in \Omega$, a space of sufficiently regular (continuously differentiable) functions.

Typically, the specification of the state, input and output for a motor neuroscience problem depends on the level of abstraction of the intended theory (Valero-Cuevas et al., 2009). In this paper, we consider $\mathbf{u}(t)$ to be joint torques or actuator forces. The aim of control in the system $\mathcal{F}$ is to influence the behaviour in order to satisfy task requirements. For the scope of this paper, we simply define behaviour as to the trajectory followed by the system in accomplishing a task. A task $\mathcal{T}$ is denoted by a set of constraints that must be obeyed, i.e. by the tuple $C_{\mathcal{T}} = \{\mathbf{y}_{\mathcal{T}}(t_d) = \mathbf{y}_{\mathcal{T}t_d}, \dot{\mathbf{x}}_{\mathcal{T}}(t_d) = \dot{\mathbf{x}}_{\mathcal{T}t_d}\}$. This can be specified by a set of boundary conditions on the behaviour such as for reaching or as a discrete set of via points to be followed.

A trajectory is then denoted by T, one of the multiple possible unique paths in the task space which satisfies all of the task constraints C_T. that satisfies all of the task constraints. We assume that multiple trajectories exist which can satisfy some task requirements. For this system, from an engineering perspective, the feedforward control problem is to compute the function $\mathbf{u}(t) = f_f(, \mathbf{x}(t_0))$. Let us denote then $u(t) \in \mathcal{U}$ as the set of admissible control inputs that satisfy the desired objectives $C_{\mathcal{T}}$. There may exist multiple solutions for the task, i.e. multiple trajectories, and therefore the cardinality of $\mathcal{U}$ could be considered to be greater than 1. This relation is the well-known *redundancy problem* of motor control, i.e. there is a non-univocal relationship between observed movements and input actuation (Bernstein, 1967).

Since motor behaviours are an important determinant of survival, it has been suggested that Darwinian principles might also underlie movement control. The hypotheses of optimal motor control therefore suggest that the solution to the redundancy problem arises from control mechanisms that are minimising some form of cost function $J(\mathbf{x}(t), \mathbf{u}(t), t)$. Typically such cost functions are justified by citing various biologically relevant factors that impact survival such as energy requirements, accuracy, stability of control etc (Hogan, 1984b; Harris and Wolpert, 1998; Harris, 1998a).

For the system in Eq. (1), we observe that the complexity of learning the control in this case is dictated by a number factors such as the dimensionality of the input $\mathbf{u}(t)$ given by N_i, the dimensionality of the goal in the output space $\mathbf{y}(t)$ given by O, the temporal complexity of the goal trajectory $\mathbf{y}_{\mathcal{T}}(t_d)$, the complexity of the cost function $J(\mathbf{x}(t), \mathbf{u}(t), t)$ and finally the dimensionality of the state x given by N. For even moderately large dimensional systems, this represents a serious limitation on the tractability of computing an appropriate control policy., i.e. the *curse of dimensionality*. Also, effects such as nonlinearities in the functions $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ can further complicate the problem.

Optimal control theory models such as the minimum energy, minimum torque change, minimum jerk, and the minimum variance all therefore may be intractable for systems with anything more than moderately large number of dimensions. Most results so far have been computationally generated only for very simple systems approximating real musculo-skeletal structures (Harris, 1998a). Clearly the redundancy and dimensionality problem is not just a motor neuroscience question but represents a constraint on learning for an organism (Kuppuswamy et al., 2012).Thus the high dimensional learning problem in an ontogenetic time scale for an organism is therefore directly related to the redundancy resolution strategy. We next introduce the temporal muscle synergy formulation in this framework.

2.1 TEMPORAL MUSCLE SYNERGY FORMULATION

Most models of the muscle synergy hypothesis tackle the DoF problem by constraining the space of control inputs into combinations of predefined primitives. The temporal synergy formulation has the advantage of conveniently delineating the spatial task-dependent and temporal task-independent components of a synergistic control (Alessandro et al., 2013). Temporal synergies have been extracted in a variety of tasks (Ivanenko et al., 2004; Chiovetto et al., 2013) and are usually cited as a direct example of dimensionality reduction in the control input (Ivanenko et al., 2005) with relevance to development and evolutionary theories (Dominici et al., 2011). In this formulation, the input $u(t)$ is constrained in the form of a weighted linear combination of S synergies $\psi_i(t)$ represented by,

$$\mathbf{u}(t) = \sum_{i=1}^{S} w_i \psi_i(t), \tag{2}$$

which can be rewritten in matrix notation by $\hat{W}\Psi(t)$ such that $\Psi(t) = [\psi_1(t) \dots \psi_S(t)]^T$ defines the temporal synergies and the weight matrix $\hat{W} = [w_1 \dots w_S]$ contains the linear combinators approximating a particular input signal $\mathbf{u}(t)$. There is a unique $\hat{W}$ for a given $u(t)$ if the functions $\psi_1(t), \dots \psi_s(t)$ are linearly independent and $\hat{W} \in \mathbb{R}^{I \times S}$, i.e. they are an orthonormal basis set of the space of inputs. Thus, the synergies are specified as a task-independent basis spanning the space of inputs, while the weight combinators are then computed in a task-dependent manner.

The control learning problem is to obtain the appropriate weights $\hat{W}_d$ corresponding to a desired task $\mathbf{y}_d(t)$. Due to the reduction in dimensionality, the desired solution is within a space of size $N_i \times S$. This is a linear space of inputs, therefore learning can be accomplished by a number of tools and superposition can be utilised to generalise to novel problems.

However, despite the reduction in dimensionality of inputs, we contend that the complexity of the optimal motor control problem may not necessarily be reduced simply through reduction of input space dimensionality. For instance, if the desired cost function is a function of the state $\mathbf{x}$, the state dimensionality is a bottleneck affecting learnability. Also, the specification of the task might have an important role to play in existing methods of quantifying the dimensionality of synergies (de Rugy et al., 2013); i.e. the number of synergies may be insufficient to ensure optimal behaviour. Futhermore, in order to facilitate

It is for this reason that we introduce the TSDA to quantify the reduction in dimensionality of the state space, if any, in using synergistic control. We utilise a system balancing approach using a Hankel singular value measure as described next.

2.2 DIMENSIONALITY REDUCTION AND HANKEL SINGULAR VALUES

From the control engineering viewpoint, the aim of dimensionality reduction is to simplify the input-output dynamics of a system in order to reduce the complexity of simulation and control optimisation. Many algorithms have been proposed for model and controller order reduction (Antoulas et al., 2001) including both analytic and computational methods. The reduction problem is stated as follows.

Consider the state-space model of a system in Eq. (1). The DR problem in the synthesis of an equivalent system given by,

$$\tilde{\mathbf{y}}(t) = h'(\mathbf{z}, t), \quad \dot{\mathbf{z}} = f'(\mathbf{z}, t) + g'(\mathbf{z}, \mathbf{u}, t), \tag{3}$$

where $\mathbf{z}(t) \in \mathbb{R}^{\mathcal{K}}$, and typically the dimensionality of the new state variable $\mathcal{K} < N$. Note that when driven by identical input signals $\mathbf{u}(t)$ the output of the reduced system is $\tilde{\mathbf{y}}(t)$ which is close to $\mathbf{y}(t)$ for some measure of similarity; the dimensionality of the inputs and outputs remain unaffected by the reduction.

We seek a quantification of DR in a system instead of simply reducing it to the form of Eq. (3). Therefore in this paper, we define the reduced dimensionality of a system by the operator $\mathcal{D}$,

$$\mathcal{D}(\mathcal{F}) = D, \tag{4}$$

where $D \in \mathbb{Z}^{+}$, the space of positive integers. For the system defined in Eq. (1), $1 \leq D \leq N$ for whatever measure of dimensionality that is employed. Obviously, $D = \mathcal{K}$ for the reduction leading to the system in Eq. (3).

In order to achieve this kind of a reduction, the commonly used approach is to compute a projection of the full dimensional system into a lower dimensional subspace. This is defined as a mapping W, such that, $\mathbf{z} = W\mathbf{x}$, such that certain conditions are met in the input, state and output relationship; various methods exist for computation of an appropriate W. We utilise the well known method of system balancing[1] (Moore, 1981) due to its relevance for control and stable numerical properties. Through system balancing, we seek to rotate the system coordinates (i.e. the state-space) in order to balance the controllability

[1] Also known as balanced reduction

(difficulty of reaching a state) and observability (difficulty of observing a state) of the system (Skogestad and Postlethwaite, 1996).

This process reorganises the system by ranking the importance of each of the state variables using a Hankel Singular Value (HSV) measure. They are defined as the square root of the eigenvalues of the product of the controllability ($\mathcal{P}$) and observability Gramians ($\mathcal{Q}$); measures computed on the dynamics of the system. Through balancing, the state space of the system is rotated in order to equalise both of these Gramians. The HSVs of the balanced system are thus of descending order of magnitude. This results in a transformation of the system to a basis where states that are easiest to reach (control) are simultaneously are easiest to measure (observe). Balanced reduction and HSVs are ideally suited for reduced dimensional control because they preserve stability of the original system and offer bounds on the approximation errors (Gugercin and Antoulas, 2004).

System balancing have numerically stable implementations and have been applied to a wide variety of systems (Antoulas et al., 2001). While analytical solutions exist in the linear case, empirical methods have been developed for nonlinear systems (Hahn and Edgar, 2002; Lall and Marsden, 2002). A short summary of obtaining the controllability and observability gramians and the balancing transform is presented in the Appendix.

In the context of the muscle synergy hypothesis, system balancing has been used by Berniker et al. (2009) in order to reduce the dimensionality of a nonlinear model of a frog's leg. However instead of just reducing the system dimensionality, we focus our analysis on using HSVs as a quantifier for the subspace dimensionality of a given system.

The HSVs are defined as the ordered set $\sigma_{HSV} = [\sigma_1, \ldots \sigma_N]$ where σ_i denotes the control "energy" of the i^{th} state variable in the sense of the controllability and observability Gramians ($\mathcal{P}$, $\mathcal{Q}$ respectively). HSVs quantify the dimensionality of the subspace of the balanced systems through truncation to first k states chosen by some criteria; thresholding is the approach we use. If the HSVs are normalised by using the sum, the DR is given by,

$$\mathcal{D}_{\mathcal{HSV}}(\mathcal{F}) = \begin{cases} \mathcal{K} & \text{if there exists } \sigma_{\mathcal{K}} \leq t_r, \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

where the threshold $t_r \in \mathbb{R}^+$, $t_r \leq 1$, and the resulting $\mathcal{K} \in \mathbb{Z}^+$, with $1 < \mathcal{K} \leq N$. Clearly, this form of DR is dependent on the choice of threshold. In the case of control engineering applications, the threshold is chosen on the basis of careful observation of the system (Antoulas et al., 2001). In our approach, presented next, we present a method to simplify choice of this threshold.

2.3 TRAJECTORY SPECIFIC DIMENSIONALITY ANALYSIS (TSDA)

Through system balancing we can quantify the DR of a system. This is a task-independent quantification and depends on the system properties, for e.g. the passive mechanical properties. However, if DR is to be utilised in order to merely facilitate learning and real-time control, the task-dependent reduction of the spatio-temporal dimensionality of behaviour must instead be considered. The principle underlying our approach is to analyse the dimensionality of the subspace of a system when the input is constrained in the form of temporal synergies.

The system in Eq. (1), utilising temporal synergies can be represented by,

$$\mathbf{y}(t) = h(\mathbf{x}, t), \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + \hat{g}(\mathbf{x}, \Psi, t), \tag{6}$$

We term this as a constrained-reformulation of the system dynamics where the inputs are the temporal synergies $\Psi(t)$, and can be viewed as signals which control the onset and termination of the movements for a task. For the duration of the behaviour, the dynamics is thus described by Eq. (6) due to the constrained input function $\hat{g}(\cdot)$ as,

$$\hat{g}(\mathbf{x}, \Psi, t) = g(\mathbf{x}, \hat{W}\Psi, t). \tag{7}$$

It must be emphasised that the constrained-reformulation only describes a '*virtual*' system dynamics for the duration of the movement when actuated by the synergistic input $\Psi(t)$. The state-space however has not changed; i.e. the state variable x for constrained-reformulated system is the same as the original system. Let us denote the system of Eq. (6) by $\hat{\mathcal{F}}(f(\cdot), \hat{g}(\cdot), h(\cdot))$.

Clearly, $\hat{F}$ is unique to a given trajectory and given synergy basis set, since it incorporates the weight matrix $\hat{W}$ corresponding to a trajectory T and uses input signals in the form of temporal synergies. We therefore consider $\hat{F}$ to be a trajectory specific constrained-reformulation of the dynamics. Then the trajectory specific dimensionality is given by,

$$\mathcal{D}(\hat{\mathcal{F}}) = D_T, \tag{8}$$

If $\hat{W}$ is computed to solve a given task $\mathcal{T}$ uniquely, Eq. (8) gives the DR of the equivalent trajectory that satisfies the task requirements. The TSDA measure can be contrasted against the intrinsic DR of the system of Eq. (4), which is task independent.

In this formulation, although any reduction can be utilised for computing $D_{\mathcal{T}}$, we use the system balancing and HSV based approach due to its relevance for the control problem. HSVs measure the importance of each of the state variables of the system $\hat{\mathcal{F}}$ or both the outputs (the task) and the inputs (synergy patterns). Thus they quantify the DR of the behaviours that is dependent on the kind of synergy patterns used.

However, it is ideally desired that the threshold dependency of the HSV measure as in Eq. (5) is reduced. Depending on the structure of the constrained-reformulated system, it can be expected that HSVs computed for different trajectories may be of completely different orders of magnitudes. Even if normalisation using the sum of the HSVs is employed, this may complicate the choice of threshold to compare trajectories. Furthermore this could limit the applicability of the method in comparing different kinds of temporal synergies in reducing the dimensionality.

In order to address this issue in our approach, we simply normalise the HSVs after utilising a cummulative sum. First the individual HSVs are redefined by,

$$\tilde{\sigma}_i = \sum_{j=1}^{i} \sigma_j / \sum_{l=1}^{N} \sigma_l, \tag{9}$$

therefore, the vector $\tilde{\sigma}_{HSV}$ is the normalised cumulative sum of σ_{HSV}. This process renders the relationship $\tilde{\sigma}_{HSV_N} = 1$. Thus, independent of basis or the weight matrix magnitude, the threshold can be chosen to lie in the interval $0 < t_r < 1$. We later discuss the implications of the choice of threshold magnitude on development.

Using the thresholded normalised HSVs, the TSDA is therefore given by,

$$\mathcal{D}_T(\hat{\mathcal{F}}) = \begin{cases} \mathcal{K} & \text{if there exists } \tilde{\sigma}_{\mathcal{K}} \leq t_r, \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

The TSDA can therefore be computed for both linear and nonlinear systems (see appendix for the equations). It must also be noted that through computation of the TSDA also an equivalent reduced dimensional model of the behaviour is also computed. For the scope of this paper, we focus on the dimensionality quantification and the implications of its minimisation, which is described next.

2.4 MINIMUM DIMENSIONAL CONTROL

The objective of this paper is to examine the facilitation of optimal learning and control due to muscle synergies. Through the approach developed in the previous section, we can compare various trajectories that satisfy task requirements in terms of the reduction of dimensionality. However, it order to understand the functionality of synergies, it is important to quantify the minimum dimensionality that is possible in order for a specified set of synergies to solve a task. This model of Minimum Dimensional Control (MDC) is developed by first defining the minimisation problem as follows.

The minimum dimensional trajectory for a given task is defined by the optimal weight matrix $\hat{W}^*_{\mathcal{T}}$ as,

$$\begin{aligned} \hat{W}^*_{\mathcal{T}} &= \underset{\hat{W}_T}{\text{argmin}} \quad J(\mathcal{D}_T), \\ &\text{subject to} \quad \dot{\mathbf{x}} = f(\mathbf{x}, t) + g(\mathbf{x}, \mathbf{u}, t), \\ &\mathbf{y}_{\mathcal{T}}(t_d) = \mathbf{y}_{\mathcal{T}t_d}, \dot{\mathbf{x}}_{\mathcal{T}}(t_d) = \dot{\mathbf{x}}_{\mathcal{T}t_d} \end{aligned} \tag{11}$$

In order to generalise our approach to different kinds of systems, a computational solution is ideally sought. Therefore, the cost function $J(\mathcal{D}_T)$ needs to be continuous, and must be computationally simple for any kind of system $\hat{F}$.

From the definition of the normalised HSVs in Eq. (9), it can be seen that $\tilde{\sigma}$ is positive real, bounded, and ordered set of magnitudes. Also, by definition, the difference between adjacent HSVs, given by $\delta = \tilde{\sigma}_{i+1} - \tilde{\sigma}_i$, always monotonically decreases towards 0. This implies that the crucial determining factor for minimum reduced dimensionality $\mathcal{K}$ is the magnitude of the second cumulative HSV $\tilde{\sigma}_2$. This is because the magnitude of subsequent HSVs will be greater while the first HSV magnitude $\tilde{\sigma}_1$ is irrelevant for the reduction since $D_{\mathcal{T}} \geq 1$.

For any convenient choice of threshold t_r, a large magnitude of $\tilde{\sigma}_2$ ensures that $\mathcal{K}$ is minimised since all subsequent HSV values $\tilde{\sigma}_{2...N}$ are in the interval $[\tilde{\sigma}_2, 1]$. Effectively, increasing $\tilde{\sigma}_2$ is equivalent to increasing the range of values of t_r that result in a reduction to a system of subspace dimensionality 1. Clearly, $\tilde{\sigma}_2$ is the critical magnitude determining reduction in dimensionality.

Based on this rationale the cost function we propose for the MDC is,

$$J(\mathcal{D}_T) = S_F(1 - \sigma_2), \tag{12}$$

where S_F is a positive rational scale factor. Computationally, the minimisation can be carried out using any convenient numerical optimisation algorithm. Since the obtained weight matrix $\hat{W}^*_{\mathcal{T}}$ is specific to a given task, a given synergy basis set and a given dynamical system, it can be expected that the obtained trajectories are similarly system, task and synergy set specific. Nevertheless, the results show that invariant characteristics similar to human movement emerge when computing MDC on some linear and nonlinear systems.

We hypothesise that MDC trajectories will lower the difficulty of task learning and optimisation. This is particularly relevant for the case of adaptive control, when the dynamics of the system changes with time and optimising schemes need to keep track of changes, i.e. necessitating a cost on the number of dimensions. The MDC proposal essentially allows task-specific adaptation which can gradually change in a manner mirroring developmental observations (Berthier et al., 1999).

It must be noted that MDC itself might be susceptible to the curse of dimensionality and is not meant to explain the neural instantiation of control signals for real time task planning and control. Instead we propose that it is a model for an optimal mechanism underlying trajectory planning in order to overcome the limitations imposed on the learnability. MDC thus represents a bridge between the muscle synergy hypothesis and the optimal motor control models of redundancy resolution.

2.5 SIMULATION SETUP

The experiments were performed on two kinds of simulated systems, (i) the linear tethered mass, and (ii) a nonlinear compliant kinematic chain.

2.5.1 Tethered mass system This system consists of a point mass constrained to move in a $2D$ plane. It is 'tethered' to an origin by weak passive forces using linear springs and is subject to visco-elastic damping. The system can be actuated by independent forces in 2 orthogonal directions, and the output describes the position in the $2D$ space relative to the origin. The dynamics of this system are described by,

$$\ddot{\phi} = -K\phi - C\dot{\phi} + \mathbf{F}_u, \tag{13}$$

where $\phi(t) = [\phi_x(t), \phi_y(t)]^T$ is the position of the mass in space, K is a stiffness matrix, C is a damping matrix and $\mathbf{F}_u(t) = [\mathbf{F}_{u_x}, \mathbf{F}_{u_y}]^T$ are orthogonal input forces actuating the system. The simulation parameters were chosen as $C = 2\mathbb{I}N/m/sec$ and $K = 6\mathbb{I}N/m$.

The system can be considered to be a simplified analogue of the oculomotor system. It describes the eye orb dynamics without taking torsional forces into consideration and approximates the passive effects of the orbital tissue. The output can be considered as the displacement angles in horizontal and vertical directions (in radians) since linear approximation of orb movements have been shown to be valid in the range of $\pm\pi/6$ radians (Bahill et al., 1980).

2.5.2 Compliant kinematic chain This system consists of a two-link planar kinematic chain with passive joint compliance; actuation is applied through the joint torques. The dynamics are described by (Spong and Vidyasagar, 2008),

$$\ddot{\theta} = M(\theta)^{-1}[N(\theta, \dot{\theta})\dot{\theta} + K(\theta - \theta_{\mathbf{0}}) + \tau], \tag{14}$$

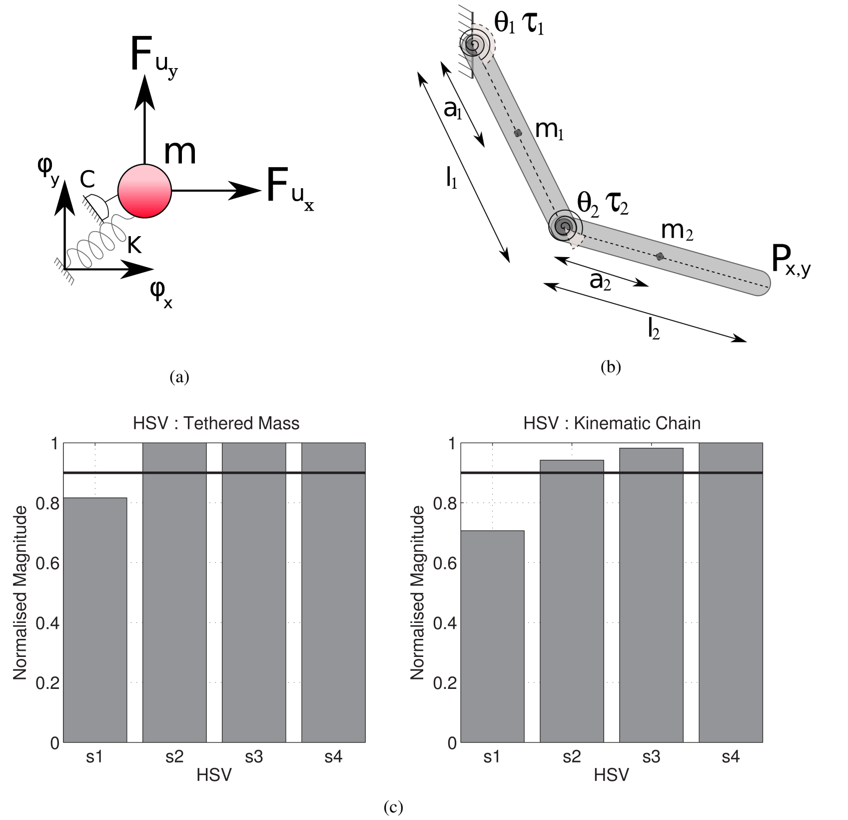

Figure 1. Physical systems employed for demonstrating the TSDA; (a) Tethered mass (Linear) : Motion of the mass is constrained to a 2D plane. The mass is anchored to the origin by weak passive forces and actuator forces are applied in two orthogonal directions. (b) Two-link planar compliant kinematic chain (Nonlinear) : End-point motion is constrained to a 2D surface. Passive compliance and damping forces act on the joints and joint torques are used to actuate the system. The state space descriptions of these systems have identical input (2), state (4) and output (2) dimensionality. (c) Comparison of the intrinsic dimensionality reduction for these two systems; the choice of threshold of $t_r = 0.9$ (black solid line) results in state-space reduction to dimensionality $\mathcal{K} = 1$ for both systems.

where the state is described by $\theta(t) = [\theta_1(t), \theta_2(t)]^T$, $M(\theta)$ is denoted the mass-inertia matrix of the system, $N(\theta, \dot{\theta})$ is the Coriolis matrix and K is the joint stiffness matrix with rest length $\theta_{\mathbf{0}}$. The system is actuated by the torques $\tau(t) = [\tau_1(t), \tau_2(t)]^T$ at the 2 joints. The parameters of the simulation are chosen as, $m_1 = 0.75kg$, $m_2 = 0.5kg$, $l_1 = 0.4m$, $l_2 = 0.4m$. The applied torques are scaled by a factor of 1.875 at joint 1 and 0.45 at joint 2 and viscous joint friction of $0.2Nm/rad$ is used at both the joints with rest angles fixed at $\theta(t_0) = [-\pi/16, \pi/2]^T$. The output of the system is the position $\mathbf{P} = [P_x(t), P_y(t)]^T$ in the $2D$ Cartesian space which can be related to the joint angles through the forward kinematics.

This system describes the behaviour of vertebrate limbs. The passive compliance ensures that empirical balancing methods can be used to compute the HSVs since a stable equilibrium posture is a necessary condition for the procedure.

2.5.3 Synergy bases In this paper, we test 2 kinds of idealised temporal synergies of orthonormal basis functions : (a) Legendre polynomial basis ($\Psi_l(t)$), and (b) Fourier basis ($\Psi_f(t)$) in order to simplify the weight learning for the analysis; they are well known approximators used for curve fitting. They are given by,

$$\begin{aligned} \Psi_l(t) &= \sum_{i=0}^{n} a_i P_i((2t - t_d)/t_d), \\ \Psi_f(t) &= a_0 + \sum_{i=1}^{n} a_i \sin(i\omega t) + b_i \cos(i\omega t), \end{aligned} \tag{15}$$

respectively, where t_d is the duration of the movement and the corresponding weights are thus given by $\hat{W}_l = [a_0, \ldots a_n]$, and $\hat{W}_f = [a_0, a_1, \ldots a_n, b_1, \ldots b_n]$. The Legendre polynomials were computed using the standard Rodriquez formula; since the polynomials are defined in $[-1, +1]$, they are shifted to accommodate the entire duration of the intended movement.

These synergies have another convenient property that their magnitudes are bounded, i.e. abs$(\Psi(t)) \leq 1$. This property is essential for nonlinear TSDA using empirical balancing since the method involves perturbing the inputs using unit impulse signals (Lall and Marsden, 2002).

2.5.4 Simulation Framework The simulation was performed on Matlab 2012. The equations were integrated using the *ode15s* solver in th ODE package with the settings of absolute tolerance = $5e^{-2}$ and relative tolerance $1e^{-3}$. The weights $\hat{W}$ for the TSDA benchmark tasks and the MDC initialisation were acquired by using a least-squares method. The numerical optimisation of MDC was carried out using the *fmincon* routine, with the *interior point* algorithm.

3 RESULTS

The results of the experiments on the two systems using TSDA and MDC are presented in this section.

3.1 TSDA ON THE TETHERED MASS

A set of four benchmark trajectories, denoted by $T_i = \phi_i(t)$, were compared using TSDA for the tethered mass system. Each trajectory described a motion from the origin to a target output position of $[0.5, 0.5]$, each thus representing a solution to the reaching task. The trajectories were specified by via-points in Cartesian space and cubic-splines fit was computed with smoothness conditions enforced at the boundaries (2^{nd} order boundary conditions set to 0). The corresponding weight matrix $\hat{W}_i$ for the control of each of the trajectories were computed using a least-squares fit of the corresponding inverse dynamic control signals $u_{d_i}(t)$.

The result of controlling the system along the four benchmark trajectories due to Fourier basis synergy control (order 4) can be seen in Fig. 2a. The temporal synergy patterns are composed of 9 components corresponding to the sinusoidal and co-sinusoidal parts of the Fourier basis as seen in Fig. 2b. The result of the weight training can be seen in the Hinton diagram of the weight matrix in Fig. 2c. The weights, represented by the size of the shaded ellipses clearly capture the temporal components of each of the trajectories. For instance, the weights corresponding to trajectory T_1 are identical in both rows. However, the other trajectories result in contrasting contributions of the sinusoidal (columns $2-5$) and co-sinusoidal (columns $6-9$) components. Although the basis order is 4, each of the Fourier basis requires 2 parameters thus giving a weight matrix of size 2×9. For each trajectory, the constrained- reformulated system was constructed and the corresponding reduction, denoted by the vector $\mathcal{K}_T$, was computed using the linear system balancing procedure. The cumulative normalised HSVs of the constrained-reformulated system can be seen in Fig. 2d. As noted earlier, the final HSV ($\tilde{\sigma}_4 = 1$) for all trajectories T_i, i.e. the last bar in each case is always unity. The magnitude of the other HSVs reflect the task, trajectory and the synergy choice.

For this experiment, a threshold value of $t_r = 0.975$ was utilised to compute the DR (black solid lines in Fig. 2b). It can be seen that the straight line Cartesian trajectory seemingly has the minimum dimensionality of $\mathcal{K} = 1$ independent of the choice of threshold magnitude. For the chosen threshold, the DR for each of the trajectories was then obtained as $\mathcal{K}_{legendre} = [1, 3, 3, 3]$.

The procedure was repeated for the Legendre polynomial basis synergies (order 4), resulting in near identical trajectories at the output $\phi(t)$. In this case, as seen in Fig. 3a, there is a difference in the HSV magnitudes reflecting their synergy specific nature. There are minor differences between the HSV magnitudes for trajectories T_2, T_3, T_4 but they have nearly identical magnitudes for the second HSV. For the same threshold as the earlier case of $t_r = 0.975$, the Legendre polynomial basis synergies result in a different DR of $\mathcal{K}_{Fourier} = [1, 3, 3, 3]$, i.e. all trajectories except the straight line are nearly identical in dimensionality. Nevertheless it can be seen that the straight line reaching trajectory (blue color) for this task has nearly identical HSV magnitudes, allowing a DR of $\mathcal{K} = 1$ for any choice of t_r.

The MDC cost of Eq. (12) captures the difference in the DR due to these two kinds of synergies as seen in Fig. 3b. The magnitudes for the first trajectory T_1 are nearly 0. This cost can thus be used as a threshold independent measure to compare different synergies extracted on the same behavioural dataset, i.e. a task specific dataset as a potential validation method. In order to explore the results with the straight line trajectory further, the MDC experiment was performed as described next.

3.2 MDC ON THE TETHERED MASS

In this experiment, the MDC was synthesised for the tethered mass system for a point-to-point reaching task, i.e. with zero velocity at the boundary conditions. The constrained numerical optimisation computed the weight matrix for the synergies which minimise the cost in Eq. (12).

For the optimisation the initial weights were computed using a cubic spline interpolate of a trajectory fitting the boundary constraints ($\phi(t_d) = 0.5$, $\dot{\phi}(t_d) = 0$). A constraint tolerance of $\epsilon = 10^{-2}$ was used as a terminal criterion for the minimisation.

The trajectories resulting from MDC can be seen in Fig.4 for the Legendre, and Fourier basis synergies. Smooth sigmoidal trajectories emerge in both cases for multiple movement durations. The terminal cost of optimisation was obtained as 2^{nd} HSV $\sigma_2 \approx 0$ for all cases. The time normalised velocity profiles in Fig. 4b are symmetric and bell-shaped.

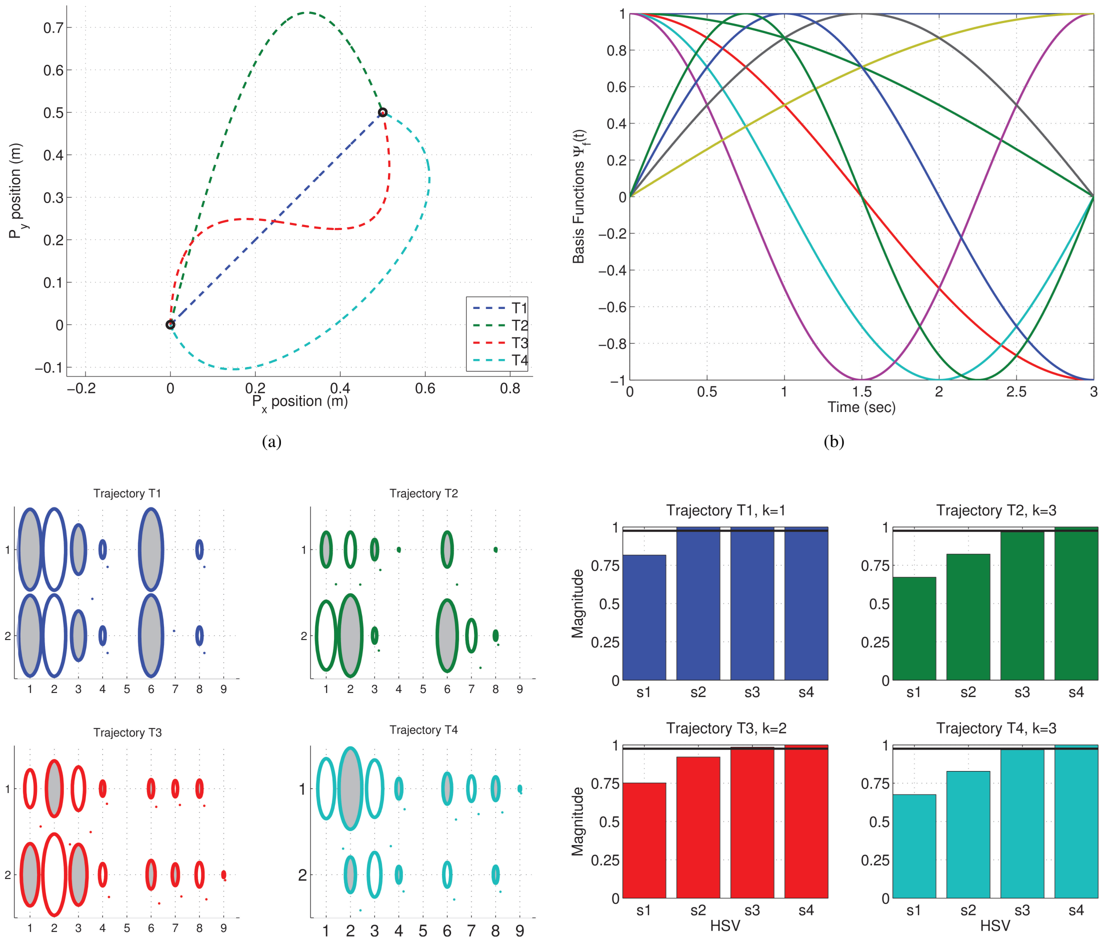

Figure 2. Trajectory Specific Dimensionality Analysis (TSDA) computed on Fourier basis synergies (order 4) actuating the tethered mass system; the task is to reach position $\phi_d = [0.5, 0.5]^t$ in a time span of $3.5sec$. (a) The four benchmark trajectories $[T_1, \ldots T_4]$ followed by the system under synergy control; (b) The Fourier basis temporal synergies used to compute control; (c) The weight matrix computed through a least-square procedure and inverse dynamics - represented as a Hinton diagram (ellipse size is the magnitude, a dark region denotes positive weight and white region denotes a negative weight). Note that the size of weight matrix is 2×9, with the rows corresponding to the 9 components necessary for a 4^{th} order Fourier basis; (d) The normalised HSV magnitudes for the reformulated composite systems for each trajectory. For a threshold magnitude choice of $t_r = 0.975$, the reformulated systems result in a DR of $\mathcal{K} = [1, 3, 2, 3]$. The system corresponding to the straight line trajectory T_1 is minimum dimensional as measured by the HSV magnitudes.

Interestingly, it can be seen that while the Legendre polynomial synergies correspond closely to the minimum jerk criterion (Hogan, 1984b), the Fourier basis synergy result is close match with the minimimum acceleration criterion (Ben-Itzhak and Karniel, 2008) (reprented by the dashed black lines

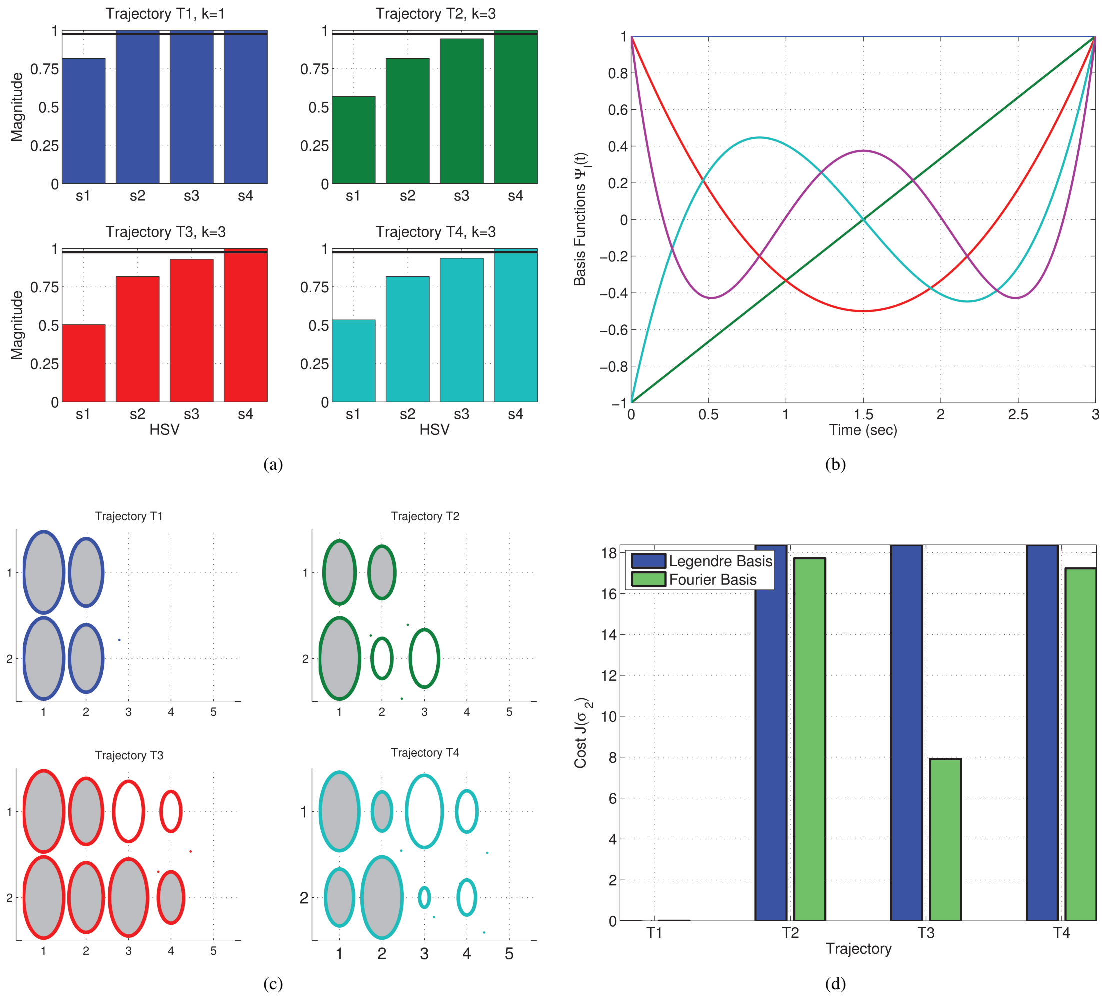

Figure 3. Trajectory Specific Dimensionality Analysis (TSDA) computed on Legendre polynomial basis synergies (order 4) actuating the tethered mass system, tracing the trajectories $[T_1, \ldots T_4]$. (a) The normalised HSV magnitudes for the reformulated composite systems for each trajectory when using Legendre polynomial basis synergies. For the same threshold magnitude choice of $t_r = 0.975$ as the previous case, the reformulated systems result in a DR of $\mathcal{K} = [1, 3, 3, 3]$; (b) the Legendre polynomial basis temporal synergies used to compute control (b) The weight matrix computed through the least-squares procedure and inverse dynamics - represented as a Hinton diagram (ellipse size is the magnitude, a dark region denotes positive weight and white region denotes a negative weight). Note that the size of weight matrix is 2×5 ; (c) A threshold-independent comparison metric of the dimensionality of the 4 trajectories - scaled normalised 2^{nd} HSV magnitude ($\hat{\sigma}_2$). The straight line trajectory results in minimum magnitude (0) of the 4 trajectories for both of these synergies.

in both cases). The velocity profiles seen in Fig. 4b also show these features. There are other minor differences between the trajectories for each kind of synergy. Nevertheless, in both cases the peak velocity of the trajectory increases linearly with the movement duration. These results show that the MDC model computes a synergy specific minimum dimensional trajectory for a given task.

Due to the linearity of the system, the weight matrix computed by MDC linearly scales with the movement duration as seen in Fig. 4c (represented only for one of the inputs). The magnitude of the changes are synergy dependent. This result implies that for linear systems the peak velocity and movement duration are a linear function of the synergy weights; the relationship depending on the synergy type.

The tethered mass system can be seen as an analogy of the human eye mechanism. The passive forces acting on the mass are similar to the weak passive forces due to the orbital tissue. Although the notion of synergies does not seem to extend to the oculomotor system, the Fourier basis synergy can be viewed as a useful modelling tool for analysis of the frequency response characteristics Harris (1998b). The bandwidth problem in oculomotor control is described further in the discussions.

3.3 TSDA ON THE KINEMATIC CHAIN

The empirical balancing procedure was used to compute TSDA on a set of four benchmark trajectories $T_{1...4}$ on the compliant kinematic chain system. The arm was initialised with the angles $\theta(t_0) = [-\pi/16, \pi/2]^T$, i.e. the rest position. Similar to the linear case, each trajectory describes a motion from the initial position to an end position $[0.5, 0.2]$ in the Cartesian space. Again, the trajectories were obtained by fitting cubic splines to some Cartesian waypoints with smoothness conditions enforced at the boundaries (2^{nd} order boundary conditions set to 0), each represents a variation on the reaching task. In contrast to the linear case, inverse kinematics is used to compute the joint angle trajectories for each case; the '*down*' configuration is utilised similar to the reaching behaviours in humans. Inverse dynamics was then utilised to numerically compute the torque $\tau_i(t) = [\tau_{i_1}(t), \tau_{i_1}(t)]^T$ corresponding to each task T_i. The weight matrix was then computed for each case using a least-squares procedure.

The endpoint trajectories for the 4 cases using Legendre basis synergy control is seen in Fig. 5a. The weight matrix is represented by the Hinton diagram in Fig. 5b. From the size of the shaded ellipse, it can be seen that in all four cases, the contribution of the proximal joint is much higher. The temporal aspects of the trajectories can been seen in the relative contributions of the negative weights (ellipses with white shading). Again, the corresponding composite system was then constructed and the empirical balancing procedure was utilised to compute the approximate HSVs for this nonlinear system. Since the Legendre synergy magnitudes are bounded; the empirical gramians were computed by applying unit impulses in place of the synergies and collecting trajectories of the behaviour.

The application of empirical balancing in this framework is equivalent to activating individual and combinations of the synergies with bounded impulses; the magnitudes were chosen from a uniform distribution about an input ball of same dimension as the number of synergies, i.e. of dimension S. The HSVs corresponding to each task T_i computed by this method can be seen in Fig. 5c. The DR using a threshold choice of $tr = 0.935$ was obtained as $\mathcal{K} = [1, 2, 2, 2]$. Similar to the earlier linear example, it can be observed that the straight line trajectory with a sigmoidal profile seemingly results in minimum dimensionality of $k = 1$. This observation was examined further in the MDC experiment, which is presented next.

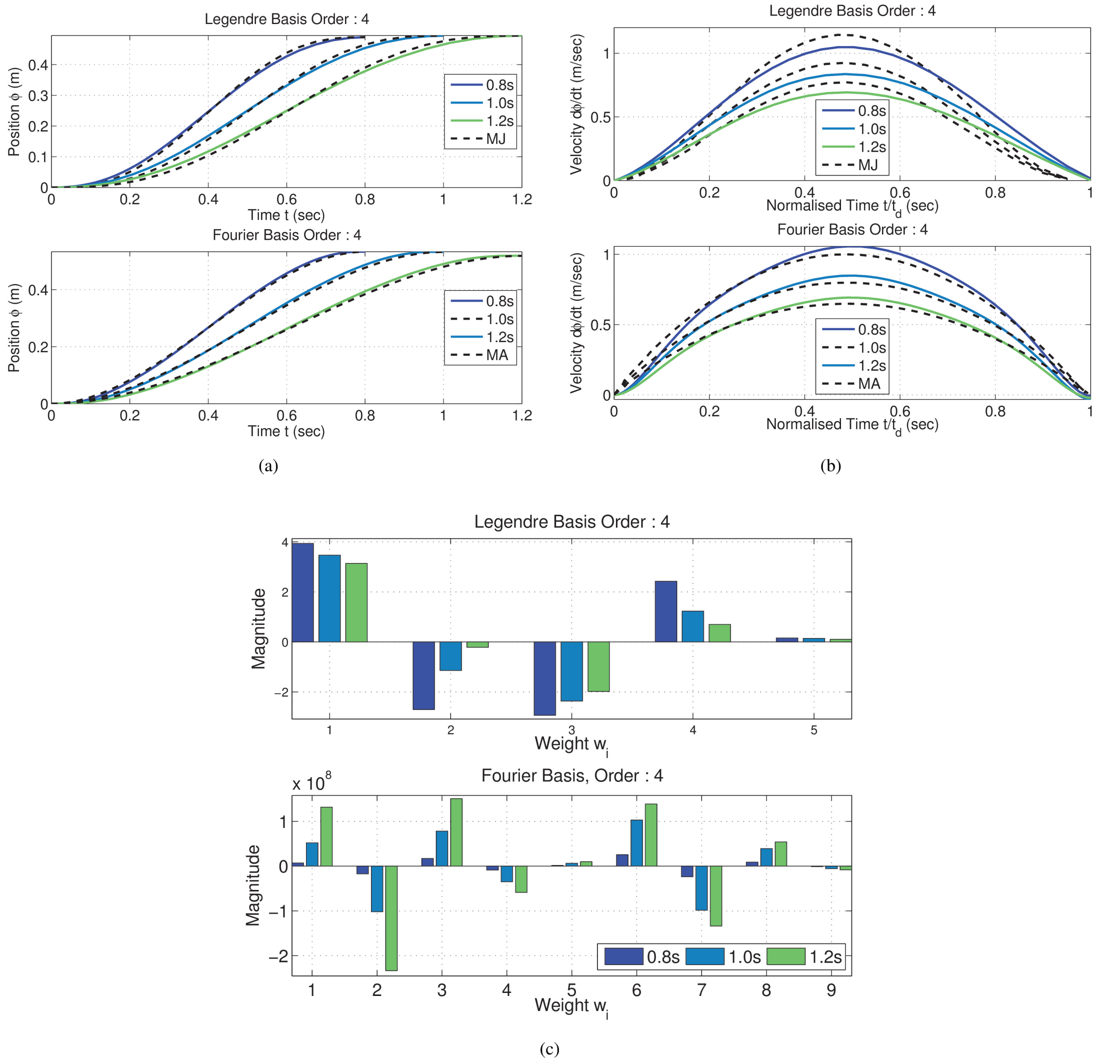

Figure 4. Minimum Dimensional Control on the Tethered Mass for reaching position $\phi_d = [0.5, 0.5]^T$ from the origin - 2 kinds of synergies (Legendre basis of order 6 and Fourier basis of order 4) and 3 desired time spans ($t_d = [0.8, 1.0, 1.2]$) analysed. Trajectory of mass traces sigmoids for all time spans and for both kinds of synergies. Trajectories (a) are similar to the Minimum Jerk (MJ) criterion for the Legendre polynomial basis and Minimum Accelaration (MA) for the Fourier basis case; (b) The corresponding bell-shaped symmetric velocity profiles. Optimum dimensional weights (d) for both Legendre polynomial and Fourier basis synergies linearly changes with increase in movement duration for each actuator.

3.4 MINIMUM DIMENSIONAL CONTROL IN KINEMATIC CHAIN

The MDC experiment was repeated on the kinematic chain system for a set of reaching targets within the workspace of the arm. Similar to the linear case, the constrained minimisation was initiated with zero

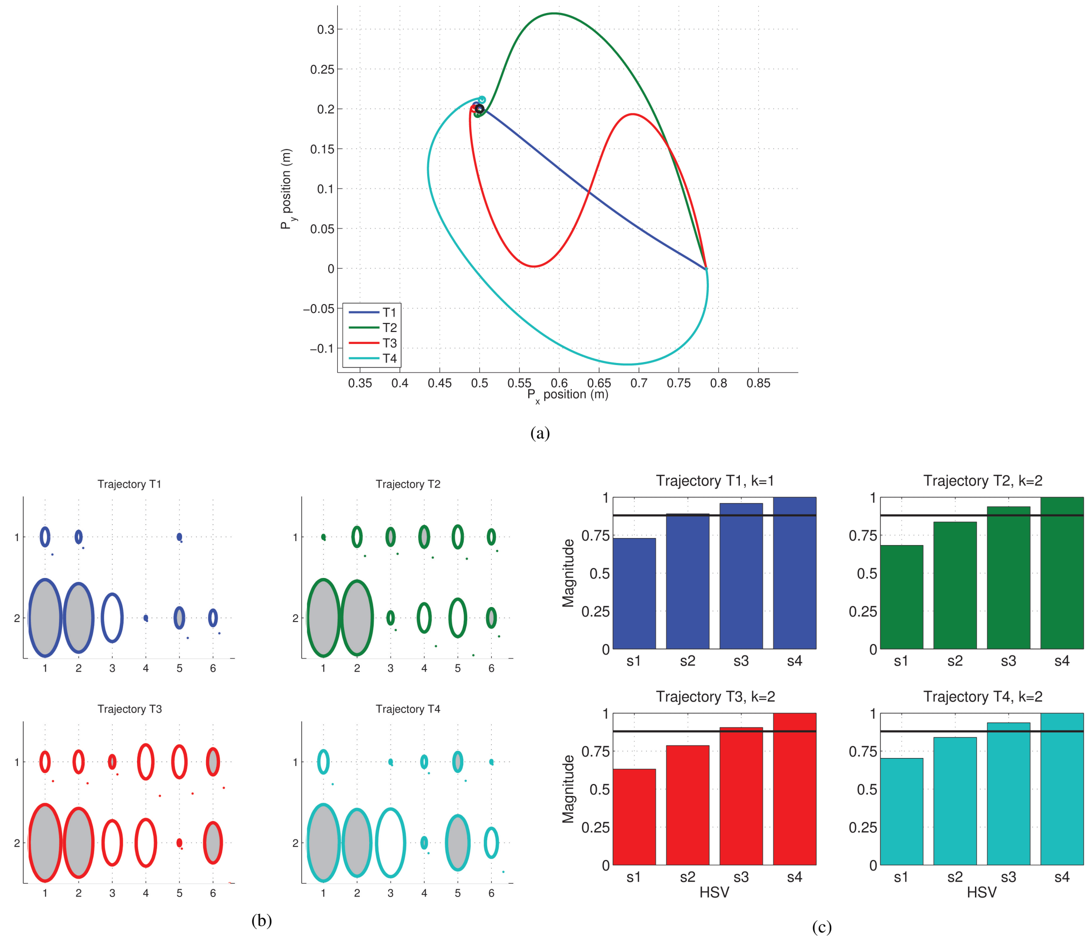

Figure 5. Trajectory Specific Dimensionality Analysis (TSDA) computed on Legendre basis synergies (order 6) actuating the compliant kinematic chain system, the task is to reach position $P_d = [0.5, 0.2]^t$ in a time span of $2.5sec$, the origin is a nearly fully extended kinematic chain. (a) Four benchmark trajectories $[T_1, \ldots T_4]$ traced by the mass under synergy control - synergy weights were computed from via-pints using a least-squares approach; (b) Hinton diagram of the weight matrix (ellipse size is the magnitude, a dark region denotes positive weight and white region denotes a negative weight).(c) The normalised empirical HSV magnitudes for the nonlinear reformulated composite systems for each trajectory. For a threshold magnitude choice of $t_r = 0.935$, the reformulated systems result in a DR of $\mathcal{K} = [1, 2, 2, 3]$ when using balanced reduction. The system corresponding to the straight line trajectory T_1 is minimum dimensional as measured by the HSV magnitudes.

velocity at the boundaries. A constraint tolerance of $\epsilon = 10^{-2}$ was used as a terminal criterion for the minimisation.

The trajectories resulting from MDC can be seen in Fig.6 for the Legendre basis synergies. Smooth sigmoidal near straight line trajectories emerge for some movement durations; the results are presented for $t_d = 2.5s$ and $t_d = 3.0$. However, in contrast to the linear system, the time normalised velocity profiles are again bell shaped but skewed in some cases. Nevertheless, the peak velocity in each axis is dependent on the movement amplitude. It can also be seen in this case that the correspondence of the obtained trajectories to the Minimum Acceleration (MA) model (Ben-Itzhak and Karniel, 2008) is greater (black dashed lines in Fig. 6a,b). Despite the skewing of the trajectories, the results suggest that the minimum dimensionality principles might underlying limb movement planning and control in humans.

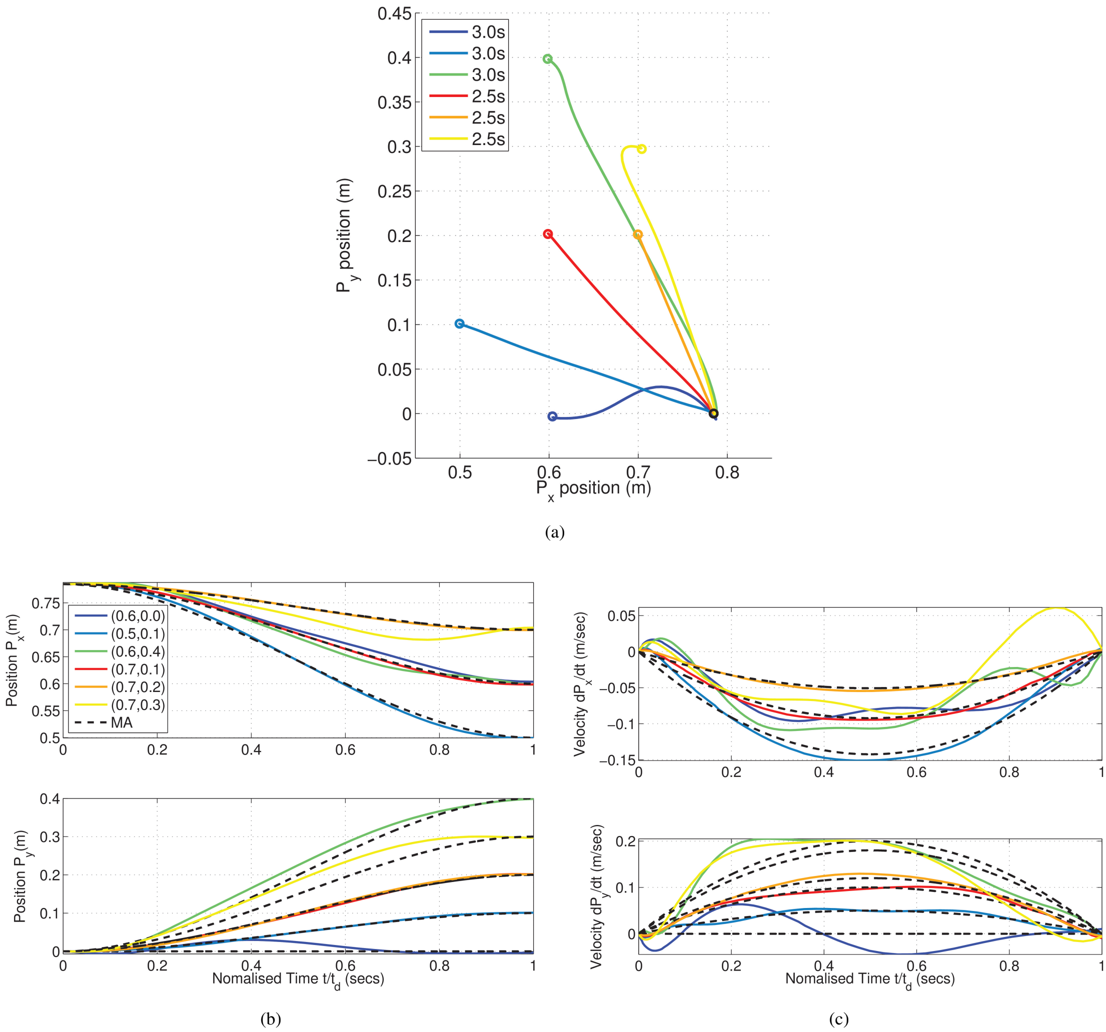

Figure 6. Minimum Dimensional Control on the Kinematic Chain for reaching various positions using Legendre basis synergy (order 7). Results show feasible MDC results for targets $[0.6, 0.0]$, $[0.5, 0.1]$, and $[0.6, 0.4]$ in a duration of 3.0 secs, and targets $[0.7, 0.2]$, $[0.7, 0.3]$, and $[0.6, 0.2]$ in a duration of 3.0 secs. Trajectory of endpoint is sigmoidal (a)velocity profiles show skewed bell shapes (b). The peaks of the velocity profiles however are close match to the Minimum Acceleration (MA) criterion result.

4 DISCUSSION

In this paper, we develop a quantification for the reduction in the spatio-temporal behavioural dimensionality in a system due to control in the form of muscle synergies. When using the temporal synergy formulation, the behaviour dynamics is dependent on the synergy bases as well as the weight matrix combining them. We model this as a trajectory-specific synergy weight constrained reformulation of the dynamics of a system. Based on the approach of system balancing, we quantify the reduction in dimensionality in the constrained-reformulated system using a threshold-normalised Hankel Singular Value (HSV) measure – the approach measures the dimensionality of the subspace of the dynamics of the balanced system. Using this approach of Trajectory Specific Dimensionality Analysis (TSDA) we show that various trajectories satisfying task constraints can be compared in terms of dimensionality a system and synergy dependent manner. Through the minimisation of this dimensionality, in our model of Minimum Dimensional Control (MDC), we obtain the weight matrix corresponding to the minimum dimensional trajectory satisfying task constrains. These methods were tested on biologically-relevant simulated linear (tethered mass) and nonlinear (compliant kinematic chain) systems. Using idealised temporal synergies, a task, synergy and system specific reduction of dimensionality of behaviour due to control using muscle synergies was demonstrated. The close correspondence of the minimum dimensional trajectories with observations of human motions suggests that the MDC principles might underlie motor control.

Bernstein's '*degrees of freedom problem*' remains a seminal observation of natural motor coordination, and continues to challenge our biological understanding as well presenting a fundamental obstacle to biomimetic engineering. Some kind of DR surely occurs, but whether it is an implicit/ emergent phenomenon (e.g. Lagrangian optimization), or an explicit 'simplifying' evolutionary and/or developmental strategy remains a conundrum. The muscle synergy hypothesis proposes that DR is a fundamental advantage resulting from the partitioning of the space of inputs (Alessandro et al., 2013). However it has faced criticism. Although statistical regularities seem to be present in the measurements of EMG, and kinematic data from subjects performing behavioural tasks, the extracted synergies are strongly dependent on the nature of observations that can be made (Steele et al., 2013). Although recent approaches for careful experiment design have aimed at addressing this criticism, the perception that this hypothesis represents only a phenomenological view of motor control seems hard to shake off (Tresch and Jarc, 2009).

Our view is that for DR to exist in biological organisms, it would need to impact on the organism's behaviour, as this is a major determinant of fitness. That is, muscle synergies would probably only evolve if they had impact on the system output and the ability to solve and learn to solve tasks. To this end, the TSDA quantifies the DR in dynamic behaviour. The dimensionality of behaviour is taken to denote the dimensionality of the state-space of the system under synergy control. It is specific to a task and to a defined set of synergies. The dynamic models obtained through the task-specific reduction of this state space are also closely related to the internal model hypothesis (Wolpert et al., 1998; Kawato, 1999). Although we do not investigate this relationship further in this work, task-specific reduced internal representations (Braun et al., 2009) are nevertheless an interesting prospect for the study of movement planning. Minimum dimensional trajectories may potentially have an important role in minimising the neural complexity required for learning task-specific internal models.

There have been some attempts have been made to fit synergy data extracted from behaviour onto musculo-skeletal models (Neptune et al., 2009; McKay and Ting, 2012; Steele et al., 2013). Our approach could thus potentially complement this analysis and allow the quantification of the differences between synergies extracted by various methods on a given dataset. It is thus a synthetic approach to testing the validity of any set of synergies in simplifying the control problem. Although we only employed fictitious synergies composed of idealised bases of Legendre and Fourier components, it can be seen that the method itself can test any synergy set specified by a time series. Moreover, TSDA can also potentially be used to test the validity of a task definition as well as the nature and quality of the number of EMG measurements that are employed for synergy extraction. Although the current proposal only tackles the temporal synergy model, it can be potentially expanded to also allow quantification of other models of synergies as well, such as the time-varying synergies (d'Avella and Bizzi, 2005).

The methods we developed in this paper represent a control-theoretic perspective on the muscle synergy hypothesis. This permits a synthetic exploration of the role of muscle synergies as facilitators of optimisation and control trough control dimensionality reduction (Berniker et al., 2009). In this view, it is not only important to extract spatio-temporal regularities from biological behaviour datasets, but also carefully examine if task control is indeed facilitated (de Rugy et al., 2013; Alessandro et al., 2013). Our approach is also similar to a recent study of the synergy hypothesis from an intermittent hierarchial control perspective (Karniel, 2013). In principle, the notion of minimal intermittancy is closely related to our concept of minimum dimensionality, and further investigation of this link is warranted.

The Optimal Control Theory (OCT) based models originate from a evolutionary perspective on behaviour; there is a fitness-driven necessity for behaviours to be optimal. Various Lagrangians have been proposed to quantify task optimality depending on the different perspectives of the system such as the output (kinematic) (Flash and Hogan, 1985), control input (minimum variance (Harris and Wolpert, 1998), minimum norm (Dean et al., 1999)), or intermediate variables (minimum torque (Nakano et al., 1999)). It must however be noted that, OCT hypotheses employ relatively complex mathematical techniques. Current theoretical limitations mean that they can analytically only be applied only on relatively simpler models such as linearised models of the oculomotor system or limb movements (Harris and Wolpert, 1998). Also, there is no testable suggestion so far as to how and where the optimisation might actually be happening in terms of actual neural mechanisms. The method proposed in this paper is possibly a step towards this goal, since we relate optimisation to the actual recruitment of synergies to accomplish tasks.

From developmental perspective, the process of acquisition of motor coordination is gradual and seemingly composed of intermediate stages of learning (Sporns and Edelman, 1993). If we consider that optimal solutions exists in a high dimensional space (system dynamics, neural control input) unique to an individual organism, then fitness must depend on the ability to find good solutions in the developmental time frame (Harris, 2011). Searching for an optimal trajectory has a little value if it takes a long time to find. We propose that the time taken to learn an optimal performance, which we call 'learnability' is itself an important parameter in a self-organising system (Kuppuswamy et al., 2012). DR is one possibility which may speed up learning, but there might be a trade-off with precision and learning rate to the extent that non-redundant degrees of freedom are eliminated. Our approach provides a mechanism to examine this hypothesis through the measurement of dimensionality of empirically measured trajectories relative to some basis set of synergies.

An interesting outcome of our approach is the emergence of smooth sigmoidal trajectories with quasi-symmetric bell-shaped velocity profiles as optimal according to our MDC criteria. The similarity at the output for two basis sets (Legendre and Fourier) as well as for linear and non-linear systems suggests the possibility of some kind of invariance at the output. Smoothness implies a potential relationship between DR and bandwidth reduction. Clearly, task demand places constraints on possible trajectories, and hence on their spectral content. In point-to-point reaching trajectories with zero velocity boundary conditions, the temporal truncation forces a strictly infinite bandwidth, with rapidly decaying spectral energy limiting envelope (Harris, 2004). The fastest movement that can be achieved without exceeding this spectral limit are the family of minimum square derivative functions, such as minimum acceleration for 2^{nd} order system, or minimum jerk for 3^{rd} order system. The DR trajectories had lower peak velocities than expected from the minim jerk profile, but were similar to minimum acceleration (dotted lines in Fig. 4, and 6). The relationship between DR and low bandwidth is unclear at present, but has two important implications.

If this invariance is upheld, it implies that the choice of basis set is not critical (presumably provided the output trajectory can be spanned by the input basis set). Indeed, it may reflect the possibility that DR occurs at the output directly. In our work we only examine the state-space dimensionality and the computation of minimum dimensional weight matrix. In principle, this approach may also be used for investigating the optimal temporal characteristics of the basis set themselves. For example, using the Legendre polynomial basis, we observe a reduction in dimensionality across tasks, both in the input as well as in the output. In this respect, it is interesting that low bandwidth signals also have low Shannon numbers (although the Shannon number is an imprecise measure of signal dimension when duration is finite).

Second, there is a coincidence between low dimensionality and optimal control. That is, if low dimensionality is maintained, optimal or near-optimal trajectories are automatically generated for a given set of boundary conditions, and the curse of dimensionality is largely circumvented. An alternative is that the optimality approach itself is a misconstrued attempt to explain low dimensionality via a Lagrangian. However, for the minimum variance model, it would be difficult to explain the known presence of signal-dependent noise unless the noise is somehow a product/compensation for DR.

This last point is also relevant to synthetic (robotic) systems. Minimising biologically relevant Lagrangians in synthetic systems does not necessarily lead to biologically realistic behaviour, but depends on the synthetic architecture. For example, minimising reaching time in a natural system appears to be achieved by the smooth bell-shape velocity profiles, but in a linear robot the same Lagrangian (functional mimicry) would be optimised by bang-bang control leading to skewed velocity profiles. In any case, finding such solutions in real-time is non-trivial, and often natural behaviour must be programmed explicitly into the artificial system (aesthetic mimicry) (Harris, 2009). However, when we consider dimensional reduction as the underlying principle for generating natural behaviour, we envision that functional mimicry in a robot would produce similar or the same natural behaviour. It is not entirely clear at present, how precisely the mimicry would need to be. It is plausible that only crude approximations are needed. A related application would be to optimise behaviour in artificial systems that are driven by pattern based mechanisms such as Central Pattern Generators (CPG) (Ijspeert, 2008). Our approach is thus a potential path towards robots with neurally inspired motor control of reduced complexity.

DISCLOSURE/CONFLICT-OF-INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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APPENDIX : HANKEL SINGULAR VALUES AND BALANCING

The HSVs are defined using the controllability and observability Gramians ($\mathcal{P}$ and $\mathcal{Q}$ respectively) for which analytical formulations exist in the linear case. A Linear Time Invariant (LTI) system can be described in the form of Eq. (1), defined by $f(\mathbf{x}, t) = A\mathbf{x}(t)$, $g(\mathbf{x}, \mathbf{u}, t) = B\mathbf{u}(t)$, and $h(\mathbf{x}, t) = C\mathbf{x}(t)$, i.e. the matrices $[A, B, C]$. then the controllability $\mathcal{P}$ and observability $\mathcal{Q}$ grammians are defined by,

$$\mathcal{P} = \int_0^\infty e^{At}BB^Te^{A^Tt}\mathrm{dt}, \quad \mathcal{Q} = \int_0^\infty e^{A^Tt}C^TCe^{At}\mathrm{dt}, \tag{16}$$

These grammians allow quantification of how controllable and how observable the state variables are; taken together they measure the "importance" of individual state variables and can thus be used for a dimensionality reduction algorithm. The Hankel Singular Values (HSV) of a system σ_{HSV} are obtained from,

$$\sigma_{HSV} = \sqrt{\lambda(\mathcal{PQ})}, \tag{17}$$

where λ refers to the eigenvalues, and the set $\sigma_{HSV} = [\sigma_1 \ldots \sigma_N]$ are the HSVs corresponding to each state variable.

The HSVs can be viewed as a score of the control "energy" of the state variables. Thus to reduce dimensionality it is sufficient to eliminate the states with a low HSV magnitude. This step is automated by obtaining a rotation on the system T of the form $\hat{\mathbf{x}} = T\mathbf{x}$ which reorders the states in decreasing magnitude of HSV. This procedure is known as balancing. Computational efficient methods exist for linear systems for computing the balancing transform T (Laub et al., 1987). Then its possible to truncate the resulting system to the first $\mathcal{K}$ states - a technique known as Balancing Truncation (Moore, 1981). The choice of $\mathcal{K}$ is thus left to the control design and is usually fixed after examination of the HSVs (Hahn and Edgar, 2002). The method proposed in this paper uses a threshold measure on the weighted normalised HSVs to decide the dimensionality.

EMPIRICAL GRAMIANS

For nonlinear systems however, there is no general approach to compute solutions although a method based on energy functions exists in some cases (Scherpen, 1993). However, such approaches are computationally difficult and obtaining an analytical closed form expression is not guaranteed. The alternative is to employ *Empirical Gramians*, which are computed using datasets of system behaviour (Lall and Marsden, 2002).

First the system is perturbed in r different (input) directions (defined by the set $T^{n_i} = \{T_i, \ldots, T_r\}$, where ${T_i}^T T_i = I$) at s different sizes of perturbations in each direction (defined by the set $M = \{c_1, \ldots, c_s\}$ where $c_i > 0$) across all the n_i inputs and across all n states (defined by the set of unit vectors $E^n = \{e_i, \ldots, e_n\}$) of the system. Then the empirical Gramians are obtained from the resulting state trajectories as,

$$\hat{\mathcal{P}} = \sum_{l=1}^{r} \sum_{m=1}^{s} \sum_{i=1}^{p} \frac{1}{rs{c^2}_m} \int_0^\infty \Phi^{ilm}(t)dt, \quad \hat{\mathcal{Q}} = \sum_{l=1}^{r} \sum_{m=1}^{s} \frac{1}{rs{c^2}_m} \int_0^\infty T_l \Psi^{lm}(t) {T_l}^T dt, \tag{18}$$

where for the controllability Gramian $\hat{\mathcal{P}}$, $\Phi^{ilm}(t) = (\mathbf{x}^{ilm}(t) - {\mathbf{x}_0}^{ilm})^T$, for $\mathbf{x}^{ilm}(t)$ being the state of the nonlinear system corresponding to the impulse input $\mathbf{u}(t) = c_m T_l e_i \delta(t)$ and for the observability Gramian $\hat{\mathcal{Q}}$, ${\Psi^{ilm}}_{ij}(t) = (\mathbf{y}^{ilm}(t) - {\mathbf{y}^{ilm}}_0)^T(\mathbf{y}^{ilm}(t) - {\mathbf{y}^{ilm}}_0)$, and $\mathbf{y}^{ilm}(t)$ is the output of the system for the initial condition $\mathbf{x}(0) = c_m T_l e_i + \mathbf{x}_0$, and ${\mathbf{y}^{ilm}}_0$ is the steady state output. A detailed description of the nonlinear balancing model reduction utilising the empirical Gramian method can be found in (Hahn and Edgar, 2002).

QUALITY OF REDUCTION

Dimensionality reduction of a system always results in some kind of loss; it is important to quantify this loss for a reduction algorithm. For the LTI system, the closed loop transfer function is given by, $G(s) = C(sI - A)^{-1}B$, and the following guarantees exist Antoulas et al. (2001),

$$\begin{aligned} \sigma_{k+1} &< {\|G - G_k\|^2}_\infty, \\ {\|G - G_k\|^2}_\infty &< 2(\sigma_{k+1} + \ldots + \sigma_n), \end{aligned} \tag{19}$$

and barring some conditions, the stability is preserved. Although the method of Balancing does not obtain an optimal reduction in the sense of eq.(19) (an optimal reduction might exist), when there is a large drop off in HSVs, the resulting condition of the norm in eq.(19) is extremely small.

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Appendix E

Dynamical Movement Primitives and Reduced Dimensionality

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Dynamical Movement Primitives and Reduced Dimensionality

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Abstract Elucidating the mechanisms underlying motor coordination in the realisation of movements is important for both understanding natural behaviour and control design of artificial systems. Dynamical Movement Primitives have been proposed in this context as a control architecture and modelling tool composed of a programmable pattern generators to encode and replay trajectories. This paper presents an analysis of the dimensionality reduction properties of the DMP whilst controlling a linear system. First, a task-specific reformulation of the controlled system. The dimensionality of the resulting reformulated system is then analysed using Hankel Singular Values, and reduced dimensional controllers are synthesised using the technique of linear balancing. Two kinds of simulated systems, a spring mass chain and a compliant pendulum robot system are then utilised to demonstrate that there is an increase in the percentage of reduction with increase in the dimensionality of the system under control; the reduced dimensional models synthesised compare suitably to the full dimensional models in control performance. The task specific formulation and its dimensionality reduction allows comparison of tasks (trajectories) in terms of dimensionality; this is demonstrated with three kinds of benchmark trajectories. The results show that the DMP can be an effective tool for not only encoding movements but also for decreasing the dimensionality of the controlled system apart from comparing trajectories.

Keywords Dynamical Movement Primitives, Reduced Dimensional Control

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1 Introduction

One of the key open problems in the study of the motor control of organisms is the question of how coordination is achieved in nature. A fundamental challenge that a developing organism is confronted with, is the issue of coping with the large redundancy in the musculo skeletal structures, while realising a rich and varied set of behaviours; the resultant dimensionality could potentially render the learning and optimization of motor control intractable - the infamous *curse of dimensionality* (Bellman, 1961). In the context of motor control, this is often referred to as the Bernstein's *Degree of Freedom* problem (Bernstein, 1967). A significant amount of research (Latash, 2008) has been devoted to understanding the mechanisms by which the nervous control systems alleviates this problem in task learning and performance.

A consensus that seems to be emerging among motor neuroscientists is that neural control mechanisms seem to employ reduced dimensional linear control strategies, utilising the so-called *motor primitives* or also called muscle synergies in some contexts (Bizzi et al, 1991; d'Avella et al, 2003). A significant body of evidence points to such mechanisms being a part of the central nervous system (Hart and Giszter, 2010) of both, vertebrates(Mussa-Ivaldi et al, 1994), and invertebrates (Flash and Hochner, 2005) and can be utilised in a adaptive control strategy (Thoroughman and Shadmehr, 2000). A key factor that underlies these approaches seems to be the reduction in dimensionality of control; this has motivated synthesis models for primitives that utilises a reduced dimensional representation of the musculo-skeletal system (Berniker et al, 2009). Another related notion in the context of rhythmic and periodic behaviours is that of central pattern generator mechanisms (Ijspeert, 2008) that are also finding applications in the control architectures of locomoting robots.

In a robotics context, attempts have been made to synthesise and deploy a motor primitive based control strategy [to be cited], although significant drawbacks exist in that the primitives are usually hand crafted by the engineer and typically do not generalise to the various tasks. In this context, Dynamical Movement Primitives (DMP) were proposed (Schaal et al, 2003,0; Ijspeert et al, 2003) as a planning and control architecture originally for human movement imitation (Ijspeert et al, 2001). Employing a dynamic systems approach for motor planning, DMP models the plan as *attractor dynamics* modulated by a set of weighted basis functions; these weights are appropriately optimised by a learning framework for various tasks. The principle feature that makes them an attractive choice for encoding motor primitives in artificial systems, is that the planning of tasks is carried out in a linear space. They have been proposed as a unified framework unifying the dynamical systems and optimisation approaches for learning of motor control (Schaal et al, 2007). Furthermore, recent works point to its applications as a modelling tool for human motor behaviour analysis (Ijspeert et al, 2013).

Although DMPs are an excellent tool for the learning and storage of motor plans, a key question that is not yet clear is the controller's effective contribution to dimensionality of the system. Although robust techniques for training the DMP for a given trajectory exist (in a supervisory context) (Schaal and Atkeson, 1998), the question of dimensionality becomes critical in adaptive or online learning contexts (Buchli et al, 2011).Furthermore the quantification of net dimensionality, and in particular that of various encoded trajectories could serve as a very useful metric to guage the progress of motor learning (Mitra et al, 1998) in a subject.

In this paper we present a mathematical framework by which we can analyse the dimensionality of a physical system driven by DMP controllers which have been trained on various tasks. The framework relies on reformulating the mechanical system driven by DMPs into a new composite input-transformed system containing the weights where the new inputs are individual patterns in time, i.e the contribution of the basis functions. Based on this task-specific reformulation, the technique of *Balancing* (Moore, 1981) is utilised to rotate the plant state variables into a convenient coordinate system wherein the contribution of each of the state variables of the system is quantified using the *Hankel Singular Values* (HSVs). Linear systems are chosen since analytical solutions exist for the HSVs and balancing (see Appendix) and fast implementations are available [to cite?].

The system is trained for various tasks (trajectories) and the dimensionality of the resulting input-transformed system is measured for each of the cases. Reduced dimensional models are synthesised for each trajectory using a linear projection of the state space; the reduced order models are compared against the full order systems in each case on the basis of dimensionality and ability to match a predefined benchmark trajectory.

The framework allows comparison of tasks in terms of dimensionality and this is presented for a number of cases. Experimental demonstrations of the technique are provided for 2 kinds of simulated linear systems, the first being a finite length chain of masses interconnected by springs and dampers, and the second being a simulated variant of a tendon driven compliant robot platform.

This paper is organised as follows. Section 2 presents the proposed reformulation of the DMP by which the dimensionality is evaluated. The dimensionality reduction method and the measure for reduced dimensionality are introduced in Section 3. Section 4 presents the 2 kinds of models which are utilised along with a description of the experiments that were performed. The results are presented in Section 5 followed by the conclusions and discussions in Section 6.

2 Dynamical Movement Primitives : Reformulation

DMPs are learnable nonlinear dynamical systems which encode trajectories (Schaal et al, 2007; Ijspeert et al, 2003). DMPs allow movement plans to be encoded and reproduced with a set of parameters, that can be learned using regression based methods (Schaal and Atkeson, 1998). The DMP architecture consists of controllers based on tunable nonlinear dynamical systems, and can be programmed to learn complex, discrete or rhythmic, movements from a training trajectory. The controllers can be considered to be discrete or rhythmic pattern generators which can replay and modulate the learned movements, while being robust against perturbations. The switching between discrete and rhythmic pattern is accomplished by switching the canonical dynamical system which is driven by a virtual time/phase state variable. For the work in this paper, we will utilise the discrete movement formulation which utlses a damped harmonic oscillator as the dynamical system. The problem of coordinating multiple dofs in an autonomous manner is accomplished using an additional *virtual time* state variable that evolves as a $1D$ (or $2D$ for some implementations) first order system.

[Describe DMP using a simple diagram and example]

A system of n DMPs (to control n DoF) is described by n 2^{nd} order forced and damped harmonic oscillators interconnected by the state variable χ acting as virtual time variable, as described in the following set of equations,

$$\tau \ddot{z}_j = \alpha_z(\beta_z(g_j - z_j) - \dot{z}_j) + f_j(\chi), \qquad \tau\dot{\chi} = -\alpha_\chi \chi, \quad (1)$$

where, the output of the j^{th} DMP is z_j and α_z, β_z, α_χ are task independent constants, g_j is the goal position. The time constant τ is used to modulate the duration of the learned trajectories. The forcing function $f_j(x)$ is defined by a weighted and normalized summation of Gaussian basis functions,

$$f_j(\chi) = \frac{\Sigma_{i=1}^N \Psi_i(\chi) w_{ij}}{\Sigma_{i=1}^N \Psi_i(\chi)} \chi(g_j - z_{0j}),$$
$$\Psi_i(\chi) = \exp\left(-\frac{1}{2\sigma_i^2}(\chi - c_i)^2\right),$$

where, the constants σ_i, and c_i are chosen to appropriately distribute the basis function over the entire trajectory. In this formulation, the training process aims to obtain the appropriate values of w_{ij} to suitably mimic a desired trajectory. Locally weighted regression (Schaal and Atkeson, 1998) was employed for learning the weights.

2.1 DMPs and System Dynamics

DMPs are typically employed to encode kinematic trajectories in target systems (Ijspeert et al, 2013). Demonstrations have been presented using upper body humanoid robot with electric and hydraulic actuation (Schaal, 2006), flying robots (Perk and Slotine, 2006), walking bipedal systems(Nakanishi et al, 2004) etc. Although the DMP itself could encode trajectories as motor commands (torques or voltages). However many of the advantages of invariance in the DMP architecture are no longer applicable. In the case of complex robots, the output kinematic signals are converted to joint torques using turned feedforward or inverse dynamic controllers.

Consider an n link robot system with dynamics given by,

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u, \tag{2}$$

Where q is the joint vector of the robot, and $M(q)$ is the mass matrix, with $C(q,\dot{q})$ the coriolis matrix and $G(q)$ the gravity matrix; the robot driven by the joint torques u. In which case the approach of feedback linearization (Spong and Vidyasagar, 2008) is to introduce a control of the form,

$$u = M(q)\ddot{q}_d + C(q,q)\dot{q} + G(q), \tag{3}$$

for a desired acceleration $\ddot{q}_d$. The linearization can then be exploited for a kinematic trajectory by incorporating state feedback using q and $\dot{q}$ into $\ddot{q}_d$ by,

$$\ddot{q}_d = -K_0 q - K_1 \dot{q} + u, \tag{4}$$

which results in a closed loop system with the dynamics,

$$\ddot{q} + K_1\dot{q} + K_0 q = u, \tag{5}$$

this is linear dynamical system. Let us use the substitution of $x = [q, \dot{q}]^T$, this yields a matrix equation form of,

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix} = \begin{bmatrix} I & 0 \\ K_0 & K_1\end{bmatrix}\begin{bmatrix} q \\ \dot{q}\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} u \tag{6}$$

i.e. a linear system which can be written in a generic form for the entire mechanical system as,

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{7}$$

where A, B, and C are called the *state*, *input* and *output* matrix respectively. Note that x in this linear systems is not the same as χ, the canonical variable in the DMP and the output in this case y is composed of q, the joint angles. The total number of DMP's that are required is therefore dependent on the input dimensionality, which in this case is $u \in \mathbb{R}^I$

2.2 DMP reformulation

In order to understand the effective dimensionality when a a linear system is actuated using DMPs, we need to rewrite the dynamics. The approach employed in this report is to obtain the solution of the DMP in a form in which it can be used to redefine the "inputs" to the system.

The solution of the 2^{nd} order dynamical system in Eq.1 is composed of the sum of the complementary and particular solutions as follows,

$$z(t) = z_c(t) + z_p(t). \tag{8}$$

In the DMP the components can be interpreted as, $z_c(t)$ responsible for the convergence behaviour of the DMP while $z_p(t)$ encodes the trajectory with the time basis. The complementary part $z_c(t)$ can then be obtained as,

$$z_c(t) = C_1 e^{(\lambda_1 t)} + C_2 e^{(\lambda_2 t)},$$

where, $\lambda_{1,2} = -\frac{\alpha_z}{2\tau} \pm \frac{1}{\tau}\sqrt{\alpha_z^2 - 4\tau\alpha_z\beta_z g}$. The particular solution $z_p(t)$ obtained by the method of variation of parameters is of the form,

$$z_g(t) = \frac{-e^{\lambda_1(t)}}{(\lambda_2 - \lambda_1)} \int e^{-\lambda_1 t}\hat{f}(t)dt + \dots$$
$$\frac{e^{\lambda_2(t)}}{(\lambda_2 - \lambda_1)} \int e^{-\lambda_2 t}\hat{f}(t)dt,$$

where the function $\hat{f} = f(\chi) + \alpha_z\beta_z g$ and therefore the integrals are dependent on the linear summation of the DMP basis functions $f_j(\chi)$. This solution could be further evaluated, but here we offer a simplification in the form of a linear integral operator $\mathbb{L}[.]$ is employed as,

$$z_g(t) = \mathbb{L}[f(t)] \dots$$
$$+ [w_1 \dots w_m][\mathbb{L}[\phi_1(t)] \dots \mathbb{L}[\phi_m(t)]]^T.$$

The operator $\mathbb{L}[\phi_i(t)]$ is obtained from the linear sum over $f(t)$ from the solution of $\chi(t)$ as,

$$f(t) = \Sigma w_i \phi_i(t),$$

where the function $\phi_i(t) = \Psi_i(t)/\Sigma\Psi_i(t)$, expressed by,

$$\Psi_i(t) = exp\left(\frac{-1}{2\sigma_i^2}(\exp\left(-\alpha_x\frac{t}{\tau} - c_i^2\right)\right).$$

The essence of the reformulation is that the set of functions given by $\mathbb{L}[\phi_i(t)]$ represent a conversion of the DMP basis into time domain.

Separating the constants from the linear operator in Eq.**??**, we can write the full solution as,

$$\begin{aligned} z(t) =& C_1 e^{(\lambda_1 t)} + C_2 e^{(\lambda_2 t)} + \ldots \\ & [w_1 \ldots w_m][\mathbb{L}[\phi_1(t)] \ldots \mathbb{L}[\phi_m(t)]]^T \end{aligned}$$

If there are n DMPs we rewrite the system such that the functions in time are the resulting new input signal $\hat{u}(t)$ which can thus be expressed in the matrix notation as,

$$\begin{bmatrix} z_1(t) \\ \vdots \\ z_j(t) \\ \vdots \\ z_m(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{21} & w_{11} & \cdots & w_{n1} \\ \vdots & \vdots & \vdots & & \vdots \\ C_{1j} & C_{2j} & w_{1j} & \cdots & w_{nj} \\ \vdots & \vdots & \vdots & & \vdots \\ C_{1m} & C_{2m} & w_{1m} & \cdots & w_{nm} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \mathbb{L}[\phi_1(t)] \\ \vdots \\ \mathbb{L}[\phi_n(t)] \end{bmatrix} \tag{9}$$

The matrix relating $z_j(t)$ and the new inputs as functions of time, $(\hat{u}(t) = [e^{\lambda_1 t}, e^{\lambda_2 t}, \mathbb{L}[\phi_1(t)]]^T)$, is denoted by $\hat{W}$ and thus defines a new input to the linear system in Eq.7. The new inputs are depicted in Fig.1 for a 1 sec interval. Note that this may change depending on movement duration and this solution does not preserve the movement generalisation properties of the DMP system itself.

Since the constants $C_{1j}, C_{2j}, w_{1j}, \ldots w_{nj}$ are all related to the boundary conditions of the equations, which for each DMP is specified by the parameters g_j, z_{0j}. The resulting transformed linear system is expressed by,

$$\dot{x} = Ax + B\hat{W}\hat{u}, \quad y = Cx, \tag{10}$$

where the martix $\hat{W}$ is obtained as,

$$\hat{W} = \begin{bmatrix} g_{11} & z_{01} & w_{11} & \cdots & w_{i1} & \cdots & w_{n1} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ g_{1j} & z_{0j} & w_{1j} & \cdots & w_{ij} & \cdots & w_{nj} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ g_{1m} & z_{0m} & w_{1m} & \cdots & w_{im} & \cdots & w_{nm} \end{bmatrix}$$

Practically, it may not always possible to solve the equations analytically

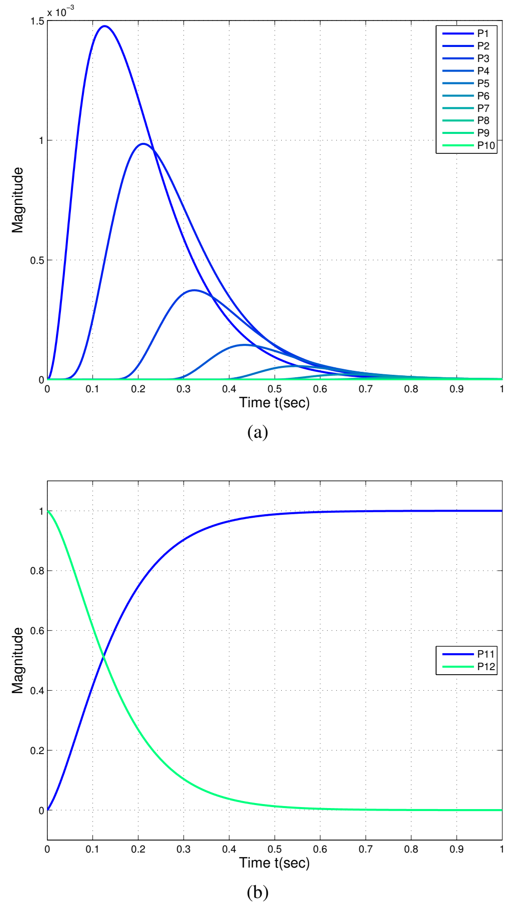


Fig. 1 Reformulated input signals for the DMP controlled system; a) The contribution of the particular solution e^{λ_1} in blue and e^{λ_2} in red, b) The individual components $\mathbb{L}[\phi_i(t)]$ for the case of 25 basis functions. All trajectories that are possible are obtained from a linear combination of each of these signals as given by Eq. 10

2.3 Iterative Basis Extraction

The approach we propose to extract the basis patterns on such systems is to iteratively utilise weight sets of the form $W_i = 1$ and $[W_1 \ldots W_{i-1}] = 0$ and $[W_{i+1} \ldots W_n] = 0$ and the solution is a linear combination which can be extracted from some sample trajectories.

For the analysis of dimensionality used in this paper, we will compare different trajectories encoded by systems of the form in Eq.10 which we refer to as the reformulated DMP (rDMP) and the original intrinsic mechanical system of the form in Eq. 7 which we refer to as the intrinsic system. In the next section, the measure of reduced dimensionality of each of these systems is presented.

3 Model Dimensionality Reduction

Often in physical systems, the behaviour is described by equations of motion derived from physical laws; these sometimes tend to be verbose in terms of number of equations, or are composed of a higher order derivatives of the state variables. Even assuming that a minimal realisation (in number of state variables) is employed in obtaining the dynamics of a system, often the order of state variables required is greater than those required from a control perspective. The aim of model order reduction or model dimensionality reduction is to obtain a lower dimensional representation of the system which is then utilised for modelling and control design.

In this context, a general class of techniques which accomplish this, is known as the *Projection* framework (Antoulas et al, 2001). The aim is to find a subspace into which the higher dimensional state variable can be projected into. The corresponding dynamics in the new lower dimensional space can be obtained by using methods such as the *Galerkin* Projection. Given a system which is described by the following dynamics (in state space formulation),

$$\dot{x} = f(x) + g(u), \quad y = h(x), \tag{11}$$

where $x \in \mathbb{R}^n$, an n dimensional space, and the inputs $u \in \mathbb{R}^i$ and outputs $y \in \mathbb{R}^o$ are such that i and o need not be equal to n. In this case, the representation we are seeking aims to find an equivalent system,

$$\dot{z} = f'(z) + g'(u), \quad y = h'(z), \tag{12}$$

where $z \in \mathbb{R}^k$, where the new state variable $k \ll n$. Note that the inputs u and outputs y do not change.

The projection Framework aims to find a reduced dimensional representation of the dynamics of a systems by "projecting" the state of the full dimensional system into a lower dimensional subspace. The aim of the projection framework is to find a mapping $\mathcal{P}$ in,

$$z = \mathcal{P}x, \tag{13}$$

such that certain conditions are met in the input output relationship, such as error bounds. The various methods approach this computation differently (Antoulas et al, 2001). A well known projection technique is the method of Proper Orthogonal Decomposition (POD)[1] which is often employed for data dimensionality reduction. The POD technique minimises systems on the basis of statistical properties of the state variables for a pre-obtained state trajectory.

However from a control viewpoint, it is more important to obtain to employ an *input-output* reduction technique which attempt to capture the input to output relation of the original system to the best extent possible while minimising the state dimensionality (**?**). The most well known of these methods is that of Balanced Reduction (Moore, 1981) which is derived from minimum realisation theory (Schutter, 2000).

Balanced reduction methods seek to obtain a rotation of the system coordinates called a balancing transform T, which maximises the importance of the state variables to the input and to the output; the individual contributions are measured by the controllability $\mathcal{P}$ and observability $\mathcal{Q}$ gramians respectively (more details in the Appendix). The technique allows the scoring of the states in terms of their importance using Hankel Singular Values (HSVs) which are obtained as a product of the 2 gramians. Closed form analytical solutions exist for linear dynamical systems, although in the nonlinear case, approximate numerical solutions have also been found (Lall and Marsden, 2002; **?**).

3.1 Measure of Reduced Dimensionality

Based on HSVs $[\sigma_1 \ldots \sigma_n]^T$ we can derive a measure $\mathcal{D}$ that characterises the number of HSVs above a threshold t_r. By this definition $\mathcal{D}$ is an integer, and $\mathcal{D} \leq \mathcal{N}$, the state dimensionality of the original system. Since we focus on the task-specific formulation of a system under control of a DMP, 2 measures can be obtained, i.e. one for the intrinsic system of Eq.7 (without DMP controller) which is task independent, $\mathcal{D}_i$, and one for the DMP controlled system that is reformulated as in Eq.10, $\mathcal{D}_d$. This criterion therefore allows the comparison of individual tasks on the basis of the measure $\mathcal{D}_d(W^*)$.

4 Simulated Systems and Experiments

The reduced dimensional behaviour of a mechanical systems under DMP control was tested on two kinds of simulated systems, first, a chain of masses interconnected by spring and damper elements, and the second, a simplified compliant tendon driven robot simulation platform based on a real robot system. While the former is a high dimensional system with output being the $1D$ position of the end mass in the chain, the latter is based on a real robot platform where the output can be considered to move in a $2D$ space.The systems and the experiments performed are described in this section.

4.1 Spring-Mass Chain System

The first mechanical system under consideration comprises of a $1D$ chain of masses interconnected by linear spring as in Fig.2a. It is fixed in one end and free to move in the other.

[1] also called Principal Component Analysis or the Karhunen Love method

Each mass element in the chain can be individually actuated by forces directly. Each element in the chain is thus subject to forces from the springs, dampers and from the input actuators. Based on this, the system dynamics are written as follows,

$$\begin{aligned} m_i \ddot{x}_i = & -k_i(x_i - x_{i-1} - l_o) \dots \\ & -k_{i+1}(x_i - x_{i+1} - l_o) - c_i \dot{x}_i + f_{ui}, \end{aligned} \tag{14}$$

where $i \in [1 \dots n]$, where $x_0 = x_n = 0$, $[m_1 \dots m_n]$ are the masses of the n elements along the chain, $[k_1 \dots k_n]$ are the stiffness of the springs interconnecting the elements, $[c_1 \dots c_n]$ is the visco-elastic damping acting on each of the elements and $[x_1 \dots x_n]$ being the positions of the elements.

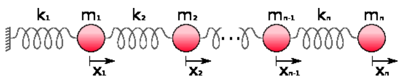


Fig. 2 Simulated test systems : A simulated one dimensional chain of masses interconnected with springs and damping elements. The fixed end is connected to the mass m_1 and the free end with mass m_n

4.2 Benchmark tasks

Three kinds of benchmark tasks (trajectories) were chosen to test the framework. These were chosen as desired trajectories $x_d(t)$ at the desired outputs of the mechanical system and were used for the DMP learning (using inverse dynamics) at the input to obtain a corresponding set of weights.

a. Exponential Cosine as in Eq.15
b. Second Order polynomial as in in Eq.15
c. Minimum Jerk Polynomial as in Eq. 15

$$\begin{aligned} x_d(t) &= x_f(1 - e^{r_1 t}) \cos(r_2 t), \\ x_d(t) &= r_1 t^2 + r_2 t + r_3, \\ x_d(t) &= x_i + (x_f - x_i) \times \dots \\ & \left(10 \left(\frac{t}{t_f} \right)^3 - 15 \left(\frac{t}{t_f} \right)^4 + 6 \left(\frac{t}{t_f} \right)^5 \right), \end{aligned} \tag{15}$$

where, r_1, r_2, r_3, x_i are individual constants chosen to fit the respective desired trajectories where the initial and final positions are x_i and x_f to be reached in a final time t_f.

5 Results

We measured the dimensionality of spring-mass chains of increasing number of elements and also that of the actuated pendulum system and also synthesised and compare the performance of the corresponding reduced dimensional models. The results are presented in this section.

5.1 Reduced Dimensionality in the Spring Mass Chain

To obtain a measure of reduced dimensionality for each of the trajectories, first the HSVs are obtained by the procedure of balancing. For the first experiment, a 4 element is studied. The normalised HSVs in Fig.3a show a significant difference between each of the trajectories, and furthermore are different from that of the intrinsic mechanical system. The corresponding reduced dimensionality is therefore $\mathcal{D}_i$ and $\mathcal{D}_d$ computed by the states below the threshold percentage t_r. For the 4 element chain $\mathcal{D}_i = 5$ and the task specific dimensionalities are $\mathcal{D}_d = 2, 3, 2$ for the exponential cosine, second order polynomial and the minimum jerk trajectories respectively.

The percentage reduction is therefore, $\mathcal{D}_i\% = 62.5\%$, and for the individual tasks, $\mathcal{D}_d\% = 25, 37.5, 25\%$ respectively for the three benchmark tasks. The variation of the percentage reduction with increase in chain length is presented in Fig. 3b. As it can be seen the percentage decrease in dimensionality shows a strong decrease with increasing length of chain.

The trajectories of the reduced dimensional systems are presented in Fig.4c for each of the 3 benchmark trajectories. It can be seen that there is a close correspondence between the reduced dimensional and full dimensional systems for each of the tasks. For these experiments, the input signals are the $u^*(t)$ from Eq. 9 for both systems. The error in state trajectory between the reduced and full dimensional systems is depicted in Fig.3. There is no significant trend observable for increasing chain length, indicating that the reduced dimensional models synthesised are sufficient in mimicking the input-output relation of the full dimensional system.

5.2 Application : Leg trajectory dimensionality analysis

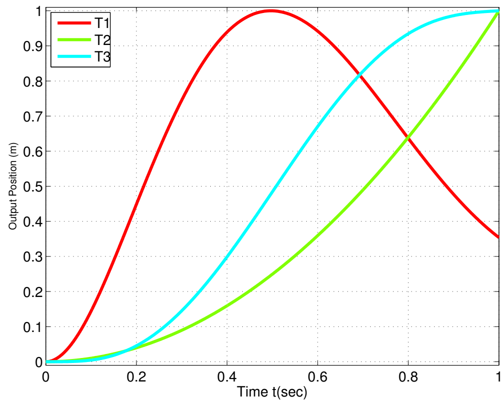
For a real world example, the approach is then used to demonstrate the effect of various trajectories on dimensionality in a nonlinear system. The robot leg system is considered to be a planar kinematic chain with joint damping. The system is as shown in fig. 5

Foot placement is often a critical aspect in legged robot motion. The results of applying the dimensionality reduction techniques are demonstrated in Fig. 6.

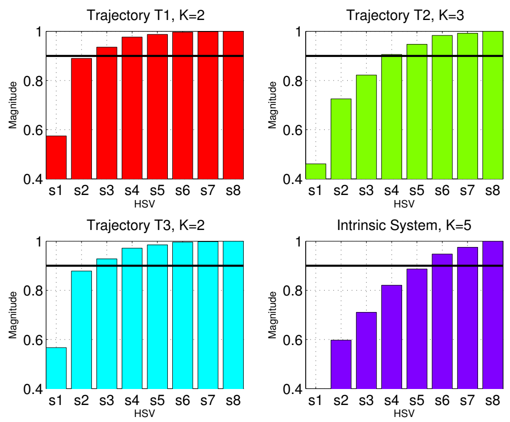
It can be seen that output trajectories that are straight lines in cartesian space have the minimum dimensionality.

6 Conclusion and Discussion

In this paper we presented a framework by which the dimensionality of a linear system under control by Dynamical Movement Primitives can be quantified. First we introduced a reformulation of a system under control by solving the

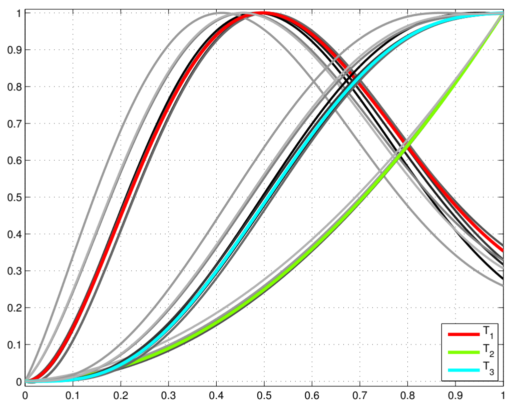


(a)

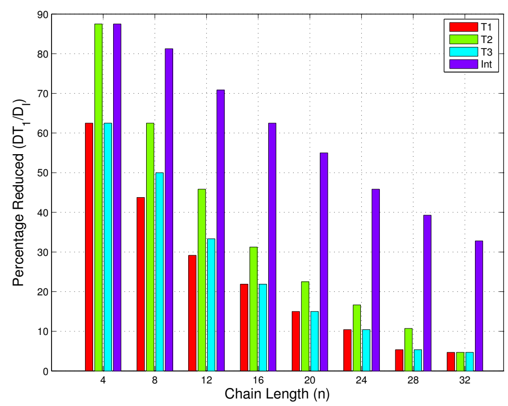


(b)

Fig. 3 Analysis of the reduced dimensionality in the spring mass chain system. a) Variation in the Normalised Hankel Singular Values for system with 2 elements (4 dimensions). The inherent dimensionality $\mathcal{D}_i$ is that of the physical system alone. Effect of increasing chain length: (b) Decrease of reduced dimensionality $\mathcal{D}_d$, (c) normalised state error (throughout trajectory) for the benchmark tasks nearly independent of chain length [TODO Correct the plot].



(a)



(b)

Fig. 4 Trajectories of the full order and reduced order systems depicted for the 3 benchmark tasks, (a) exponential-cosine, (b) 2^{nd} order polynomial, and (c) minimum jerk. The family of red lines in each case represents each of the reduced dimensional trajectories corresponding to the chain lengths of the systems analysed in Fig.3

equations of the DMP. The reformulations seperate the time domain signals from the weights by which patterns are composed. This allows the analysis of the resulting dynamical system for its dimensionality properties. The quantification of the dimensionality is done through Hankel Singular Values which measure the contribution of the state variables to controllability and observability. The system is then reduced by the process of Balanced Reduction which first rotates the system and then truncates to the states with the highest k HSVs. The resulting reduced dimensional models from the reformulated DMP system are then compared with the original full dimensional models.

Three sample trajectories are chosen to test the framework in a two kinds of simulated linear systems, i.e. a $1D$ spring mass chain and a $2D$ compliant robot simulation. The trajectories were compared in terms of dimensionality and the change of reduced dimensionality with increase of dimensionality of the original system is also measured. The corresponding reduced dimensional systems are also compared against the full dimensional systems to indicate the error characteristics of the reduced dimensional models. The results clearly show how the DMP reduced the dimensionality of the physical system in each of the cases. Furthermore, the results demonstrate that the reformulated DMP framework can be used as an effective measure of task dimensionality - a task dependent dimensionality reduction measure.

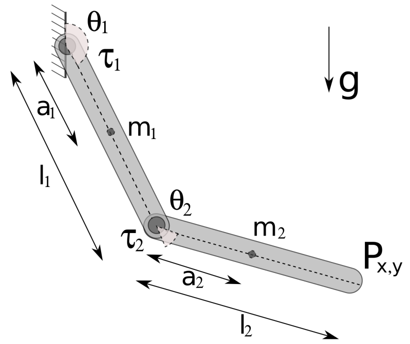


Fig. 5 A robot leg as a planar 2 link kinematic chain in gravity

The notion of reduced dimensionality is important for living systems from a learnability perspective (Harris, 2011). While high dimensionality in nature presents a creature with redundancy in solving problems and thus assist in behavioral adaptivity in the face of novel circumstances. The flipside is that learning and optimising control becomes hard or even intractable; this could be detrimental to the evolutionary need for survivability. Furthermore, real world systems need to demonstrate a high degree of adaptivity, which again is affected by high dimensionality. Reduced dimensional control strategies in the form of motor primitives (Bizzi et al, 1991) or muscle synergies (d'Avella et al, 2003) have been proposed in this context are a mechanism alleviating the so called *curse of dimensionality* (Flash and Hochner, 2005).

Inspired by the notion of linear combinations of stored pattern mechanisms, DMPs were proposed as a control architecture that has found many applications in robotics which also serving as a useful modeling tool for biological phenomena (Ijspeert et al, 2013). The underlying design principle of a tunable or programmable attractor system for encoding goal-directed behavior, is applicable for many other nonlinear dynamical systems models and could find many computational neuroscientific applications. Although convinient learning techniques have been proposed (Schaal and Atkeson, 1998) for training the weights for encoding a desired trajectory, a key question that needed clarification was if it has an impact on the systems dimensionality. For instance, could there exist a scenario that usage of this controller serves to increase the system dimensionality and thus complicate task learning rather than decrease. Although robust techniques for training the DMP for a given trajectory exist (in a supervisory context) myyciteSchaal1998, the question of dimensionality becomes critical in adaptive or online learning contexts (Buchli et al, 2011). Our framework is to the best of our knowledge the first quantification and exploration of this notion.

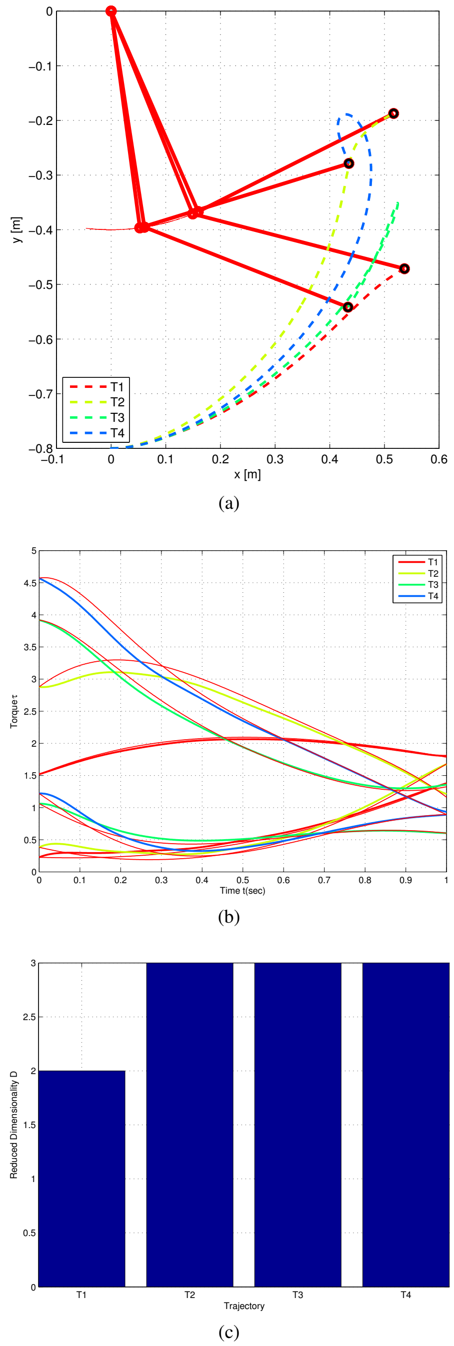


Fig. 6 Effect of DMP control on the dimensionality of trajectories in the kinematic chain

The notion of dimensionality during the process of actively learning a new task has been discussed in the context of the Bernstein problem (Bernstein, 1967). An compensatory strategy that is often discussed in motor and behavioural learning literature is that of freezing and unfreezing degrees of freedom (Newell and Vaillancourt, 2001); works in developmental robotics also have employed this mechanism (Berthouze and Lungarella, 2004). It has also been hypothesised that motor skill acquisition in a behavioural sense is an decrease of active dynamical degrees of freedom (Mitra et al, 1998). Our framework potentially allows examination of this hypothesis in human subjects by employing DMPs and motion capture mechanisms. An interesting question that can be examined is that of minimum dimensional trajectories which satisfy task constraints - an ongoing investigation.

Furthermore, since the dimensionality reduction process is dependent on the task dependent variables (or task context) the resulting reduced dimensional models are in a sense unique to a task context (Berniker et al, 2009). From a neural and cognitive perspective, higher level architectures such as MOSAIC (Haruno et al, 2001) have been proposed exploiting the idea of multiple forward and inverse models unique to each task context. The reduced dimensional models that are demonstrated within our framework could potentially be viewed from the perspective of forward models (Wolpert et al, 1998); this makes it essential to analyse the error in prediction of trajectories between the reduced dimensional and full dimensional models. Our results show that while the reduced DMP system shows significantly reduced dimensionality for some of the tasks, the errors in prediction do not substantially change. A future work in this direction could be examination of various task contexts in more complex systems.

Lastly, although our analysis focused on the linear case, this might be sufficient to analyse DMP based control since currently its employed for encoding kinematic strategies in robots which typically utilise feedback linearized controllers that are carefully tuned (Schaal et al, 2007,0,0). Furthermore in a neuroscientific perspective there is evidence indicating that linear models of the dynamics might be sufficient in some cases for analysis of the neuromuscular system (Frolov et al, 2000; Valero-Cuevas et al, 2009). Nevertheless a nonlinear analysis of the framework might also be of interest. Work on nonlinear balancing and approximate controllability and observability gramians (Lall and Marsden, 2002; **?**), and therefore approximate hankel singular values, might be a useful direction to explore as future work in for the analysis of reduced dimensional control in kinematic chain structures with neuro muscular nonlinearities.

Acknowledgments

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Appendix

Balancing Transformation and Reduction

This class of model reduction techniques first work by obtaining a balancing transformation T, which rotates the state variables on the basis of controllability and observability grammians (Moore, 1981; Antoulas et al, 2001). For a linear dynamical system defined by the triplet $[A, B, C]$ as,

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{16}$$

the controllability $\mathcal{P}$ and observability $\mathcal{Q}$ grammians are defined by,

$$\mathcal{P} = \int_0^\infty e^{At} BB^T e^{A^T t} \mathrm{dt}, \tag{17}$$

$$\mathcal{Q} = \int_0^\infty e^{A^T t} C^T C e^{At} \mathrm{dt}, \tag{18}$$

and are solutions of the Lyapunov equations,

$$A\mathcal{P} + \mathcal{P}A^T + BB^T = 0, \tag{19}$$

$$A^T \mathcal{Q} + \mathcal{Q}A + C^T C \ = 0 \tag{20}$$

These grammians allow quantification of how controllable and how observable the state variables are. For the nonlinear case approximated methods exist for obtaining the gramians (Lall and Marsden, 2002; **?**). Taken together, these 2 grammians measure the "importance" of individual state variables and can thus be used for a dimensionality reduction algorithm. One such measure is that of the Hankel Singular Values (HSV) of a system, σ_i obtained from the square root of the eigenvalues(λ) of the product of the 2 grammians as in,

$$\sigma_i = \sqrt{\lambda(\mathcal{P}\mathcal{Q})} \tag{21}$$

In order to utilise the HSV to rank and reduce the state variables in a system, we need to first "rotate" the dynamical system such that the states variables are organised in a descending order of "importance", i.e. organised by $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$, a process known as *Balancing*.

To suitably balance a system, we seek a balancing transform T that accomplishes a rotation of the form,

$$\tilde{x} = Tx, \tag{22}$$

where T is orthonormal.The balancing transform allows creation of an "internally balanced" system such that the new controllability and observability grammians, $\tilde{\mathcal{P}}$, $\tilde{\mathcal{Q}}$ obey the relationship,

$$\tilde{\mathcal{P}} = \tilde{\mathcal{Q}} = diag\{\sigma_1, \sigma_2, \ldots \sigma_n\}, \tag{23}$$

and the HSV are organised in a descending magnitude. The balanced system dynamics are then obtained from the new triplet $[\tilde{A}, \tilde{B}, \tilde{C}]$ as,

$$\dot{\tilde{x}} = TAT^{-1}\tilde{x} + TBu, \quad y = CT^{-1}x, \tag{24}$$

where $\tilde{A} = TAT^{-1}$, $\tilde{B} = TB$, $\tilde{C} = CT^{-1}$,

From this rotated system, the new reduced dimensional representation may be obtained by simply truncating the balanced system to the first k states, which can be chosen by a quality threshold. The truncated system is then obtained by expressing the balanced system matrices as,

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{25}$$

and thus obtaining the new triplet as $[A_{11}, B_1, C_1]$, where $A_{11} \in \mathbb{R}^{k \times k}$. Variations on the method include that of Singular Perturbation where the additional state variables are not truncated but are set to their steady state values [to cite].

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Appendix F

Development and Dimension Reduction

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IEEE TRANSACTIONS ON AUTONOMOUS MENTAL DEVELOPMENT 1

Development and Dimension Reduction

Naveen Kuppuswamy, Jakob Oesinghaus, and Christopher M. Harris

***Abstract*—One of the fundamental problems in developmental robotics relates to the progressive spontaneous acquisition of motor abilities by an organism. Throughout this process, the speed of aquiring abilities, which we term 'learnability', is strongly limited by the dimensionality of the sensori-motor space; this in turn could affect the survival of an organism. The existing theories on motor learning have been strongly influenced by the Bernstein notion of dimensional increase accompanying development, although counter proposals have also been suggested. The problem of redundancy resolution has also been tackled from the perspective of optimal control of motor behaviour and through theories of motor primitives, although the relationship to development is not yet clear. In this work, we present a formalisation of the dimensional change problem from the perspective of control dimensionality reduction. By utlising a projection of the neuromuscular dynamics into lower dimensional subspaces quantified by a measure called hankel singular values, we demonstrate theoritically that a progressive acquisition of skills of increasing complexity can be achieved; the change in dimensionality induced through changes both in the natural dynamics, and in the task space. As a case study, we present empirical results on dimensional change in reaching and manipulation task using a simple kinematic chain system modelling arms; the growth process resulting in gradual morphological and material property changes. The results show that there could be an optimal "path" in parameter space wherein the growth can regulate the learnability.**

***Index Terms*—Development, Bernstein Problem, Dimensional Change.**

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I. Introduction

Biological organisms (phenotypes) interact with their environments by generating physical behaviours (walking, reaching, looking, talking, etc.). These behaviours incur costs and benefits that determine the evolutionary success (fitness) of the organism's genes (genotype). Although the genotype can prepare the organism for generic scenarios, each organism will encounter unique local environments that cannot be anticipated. It is, therefore, advantageous for genotypes to imbue organisms with the ability to adjust their behaviours and morphology, which we call phenotypic plasticity. In adults, phenotypic plasticity has been extensively studied as reversible 'adaptive control' where motor commands are adapted to changes in motor dynamics, with a time-scale of hours and days in primates. Developmental plasticity is concerned with the irreversible changes that the newborn phenotype undergoes as it matures and develops, which typically takes a considerable fraction of the phenotype's lifetime. In either case, it seems likely that there should be a premium on learning as quickly as possible for a given level of competence and task complexity. The speed of learning, which we call 'learnability', depends ultimately on the dimensionality of the behavioural task space, and many have argued that dimensionality may be manipulated during development to improve learnability [refs]. Of course, any control over dimensionality must be manifest in the neural and/or structural architecture of the organisms, and must be inheritable itself (i.e. coded in DNA).

A well-known empirical observation is that organisms tend to produce rather simple and stereotyped behaviours that are frequently repeated and combined into more complex behaviours. Each simple behaviour may be modulated according to task parameters (e.g. duration, magnitude), but nevertheless clear patterns emerge, such as stereotyped arm reaching trajectories, saccadic eye movements, oscillatory walking and swimming cycles. A popular explanation for these phenomena is that they are optimal solutions to commonly encountered tasks. This makes sense in terms of maximising fitness, but optimality does not necessarily imply simplicity a priori. Moreover, it is not clear how optima can be found in high dimensional spaces (the curse of dimensionality). Another possibility is that simple behaviours are low dimensional and easier to learn.

A behavioural task is accomplished by a trajectory, $y(t)$, of effectors (hands, feet, fingers, eyes, torso, etc.). In general, there are many configurations of effectors that fulfil a task to varying degrees of success. The geometric degrees of freedom of y will depend on the number of effectors recruited for the task (M) and the orientation of each effector, so that at any point in time we need to specify O real numbers to define the physical (observable) state of the organism, hence $y \in \mathbb{R}^O$. Moreover, in general, for each degree of freedom there will be many distinguishable trajectories that can be generated depending on the duration of the trajectory, T, and the available bandwidth W. From sampling theory, the trajectory $Dy <= 2T.W.$. Thus, the total dimensionality of a trajectory is $D_T <= \sum_{k=1}^{6} M(T_k.W_k)$, which in principle could be very large.

However, biological trajectories are inherently constrained by the neuro-muscular dynamics. Consider a dynamical system of the general form:

$$\dot{x} = f(x) + g(u), \quad y = h(x), \tag{1}$$

in which, $x \in \mathbb{R}^N$ is the denoted the state, $u \in \mathbb{R}^I$ is denoted the input, $y \in \mathbb{R}^O$ denoted as the output, and the dimensionality is given by $N \in \mathbb{Z}^+$, a positive integers. where the function $f(x)$ represents the state transition dynamics (or 'natural' dynamics) and $g(u)$ is the input matrix. The output y relates to the full order state x through the output matrix C. Note that $x \in \mathbb{R}^S$, $y \in \mathbb{R}^O$ and $u \in \mathbb{R}^I$ where S is the dimensionality of the state (system order), O is the dimensionality of the output (see above) and I is the dimensionality of the input. The input reflects the activity of motor neurons, which for vertebrates can number in the thousands, but may be correlated. Here, I reflects the number of independent components. Neurons can fire at very high rates, hence the bandwidth of each input $u_i(t)$ is in principle, very high.

Although obvious, it is important to recognise that the organism can only control its task trajectory by manipulating the neural control input $u(t)$ and dynamics f, g, h. In biology and robotics, the fundamental problem is to understand how an organism can solve tasks with such high dimensionality in real-time.

A popular approach to re-cast the problem as a problem in optimal control theory (OCT) by assuming that different trajectories have different 'fitness' for the organism, and can be summarised by a scalar cost $J(y, x, u, t)$, or Lagrangian: $J = \int_{t_0}^{t_f} L(y, x, u, t)$ dt, where the interval may also be variable. The trajectory that minimises cost, $y^*(t)$, and its corresponding control $u^*(t)$ are optimal and can be found by various means ...

In general, there is no guarantee that optimal trajectory y^* can be reached by a suitable control input Development is a problem of identifying the parameters in a high dimensional space within the lifetime of the agent [9]. One strategy for alleviating the curse of dimensionality is through degree of freedom of unfeezing [20]. Task learning can however be interpreted as a decrease in active dynamical degrees of freedom [17].

II. Background

A. Bernstein and Development

The seminal research identifying motor skill learning as a problem of overcoming the hurdle of the curse of dimensionality was conducted by Russian Neuropsychologist Nikolai Bernstein [12], and published belatedly in the West in his work on movement coordination [2]. The key research problem he identified was on how the different degrees of freedom are harnessed to produce the movement form and variability associated with actions; this is the eponymous *Bernstein Problem* or the *Degree of Freedom Problem*. He viewed motor coordination as the process of mastering redundant DoF in the body and its conversion to a controllable system.

In his scheme, movement coordination aims mastering the many DoFs involved in a particular movement pattern, by reducing the number of independent variables to be controlled. Bernstein recognized that the analysis must include inertial and reactive forces along with the muscular forces since the aim of the model is not just to mime movements [25]. The movement generation must also take into account a context-conditioned variability, i.e. in the various factors and forces that act within a given task context; thus adaptivity is essential.

Bernstein proposed that task learning is a question of increase in DoF, allowing the gradual and progressive acquisition of coordination [2]. He proposed a 3 stage model of task learning :

1) Initially, reduce DoF at periphery to minimum - a process of *freezing* DoF.
2) Gradual and progressive release of DoF restrictions - a process of *unfreezing* DoF.
3) Exploring and exploiting reactive phenomena in movement control.

His model has been tested in skill learning in humans [26][19]. Even in early development there is evidence for this mechanism in the proximodistal structure in reaching in infants [3].

Although his original work was proposed with DoF being defined in a biomechanical sense, a number of works since have questioned the notion of DoF change accompanying task learning in a dynamical systems perspective [23]. There is a counter notion that task learning is a decrease in the dynamical DoF [17]. In this usage, the DoF coordination results in an equivalent dynamical system that resides in progressively fewer and fewer dimensions. There are indications that this might be the case in measurements of phase space changes during task learning [16]. An opposing view is instead that task learning is neither a decrease or increase in DoF but instead is a change induced in constraints [20] [4].

Inspite of controversies, Bernstein's ideas have been highly influential in contemporary theories of motor skill acquisition. It is still a high-level control oriented perspective on the motor control problem, especially concerning his theories on modular composition of movements. It is however not at all clear how these ideas can map to neuro-mechanical behaviour, and how it relates to contemporary theories for movement coordination such as those of motor primitives, forward-inverse model pairs, etc. This is partly due to the difficulty in modelling the neuromechanical apparatus accurately, as reviewed next.

B. Developmental Robotics

The goal of developmental robotics is to understand the biological ontogenetic development of skills and implement them in robots [1]. Intelligent behaviour is thus hypothesised to emerge through a process of self-learning in an individual and interaction with the environment. There are different kinds of skills that are acquired by such methods, the spontaneous agent-centered acquisition of motor ability is the focus of motor control development research [15], [13].

Dimensionality of the organism is relevant in the self-acquisition of a body-schema, an important cognitive concept interlinking perception and action [10]. One of the proposals for bootstrapping the motor control learning is known as motor babbling [22]. It has been noted that the associated self-organised unsupervised learning of the sensorimotor map can result in the learning of coordination through a technique of goal babbling instead [21], i.e. utilising the lower dimensionality of the task-space. This framework has also been employed in the notion of a playful machine which can deal with large body DoF through self-exploration [6]. Spontaneously acquired sensori-motor coordination relates directly to dimensionality reduction methods [24]. Furthermore, sensori-motor reduction as a natural process resulting from spontaneous activity in sleep can inspire novel developmental methods for robots [5].

An important outcome of one such developmental model, which is inspired by the Bernstein approach, is the proposal to bootstrap sensori-motor space exploration through fewer degrees of freedom; this can be followed by progressive unfreezing in later stages [14]. Such an approach can also be expanded to include alternate stages of freezing and freeing of DoF. This allows a gradual increase in task complexity that can be tackled by the robot [4]. This approach while promising is yet to be tested on high dimensional systems, and it is the view of this paper that grounding their framework on a sound theoretical basis might be the way forward.

C. Balaced Model Reduction

Balanced model reduction methods extended the minimum realisation theory of kalman to account for the controllability and observability of systems using a principal component analysis [18]. In many control problems it is sufficient to suitably model the input-output behaviour and this is best captured by the system controllability and observability gramians. Furthermore, model reduction in this paradigm can also provide insight on the causes underlying the observable dynamics of a system [8].

The method of balanced reduction first finds a transformation in the form of a rotation of system coordinates in order to "*balance*" the observability and controllability gramians; the states which have the greatest contribution to the input-output behaviour are thus obtained. Performing a galerkin projection on the most important of these states thus yields a very effective model reduction that is ideally suited for control [7]. One measure to quantify this importance is known as the Hankel Singular Value (HSV). It is computed as the square root of the eigenvalues of the product of the controllability and observability matrices, giving a quantitative "score" to the importance of each of the balanced state variables. They can thus be examined to determine the subspace to which the system must be reduced to.

Although the current state-of-art in theory for linear model reduction techniques is extensive, in the case of nonlinear systems a significant theoretical basis is currently lacking [8].

However, one family of methods that is promising for nonlinear applications is that of empirical gramians [11]. These methods combine some ideas from POD in synthesising gramians through a data-driven approach, by supplying the system with impulsive perturbations. The empirical gramians are measures that computationally obtain the standard controllability and observability gramians for the linear case, but suitably approximate them for some region of state space (defined for inputs within some bound of energies). The empirical gramian approach has been used to synthesise a model reduction technique [7] that has found successful application on some nonlinear model reduction problems.

III. Mathematical Framework

Consider the space Ω of all systems of the form (1). Formally, an element $\mathcal{F} \in \Omega$ is a tuple $\mathcal{F} = (f(\cdot), g(\cdot), h(\cdot), \mathcal{U})$, where $f : \mathbb{R}^S \to \mathbb{R}^S$, $g : \mathbb{R}^I \to \mathbb{R}^S$ and $h : \mathbb{R}^S \to \mathbb{R}^O$ are the (sufficiently regular, for example continuously differentiable) functions modelling the evolution as in 1 and $\mathcal{U}$ is the space of all allowed input functions $u(\cdot)$. Note that a triple $(\mathcal{F}, u, x_0)$, where $\mathcal{F} \in \Omega$, $u \in \mathcal{U}$, and $x_0 \in \mathbb{R}^S$, determine a *trajectory* $y = y_{\mathcal{F},u}$ of the system via

$$y_{\mathcal{F},u}(t) = h(x(t)), \tag{2}$$

where $x(\cdot)$ is the solution of the differential equation 1 with initial condition $x(0) = x_0$. Usually, we implicitly assume the initial condition $x_0 = 0$.

A task T is defined via

$$T = \{(t_i, \phi_i, \xi_i)_{i=1,\ldots,m}\}, \tag{3}$$

where $t_i \in \mathbb{R}$, $\phi_i : \mathbb{R}^O \to \mathbb{R}^k$ and $\xi_i \in \mathbb{R}^k$. Such a task is simply a list of desired conditions on the output y. More precisely, we say that an input $u(\cdot) \in \mathcal{U}$ for a given system $\mathcal{F}$ *achieves* a task T if $\phi_i(y_{\mathcal{F},u}(t_i)) = \xi_i$, i.e. the output $y_{\mathcal{F},u}$ satisfies some conditions at prescribed times t_i. An easy example would be $t = 1$, $\phi = id$, $\xi \in \mathbb{R}^O$; u achieves a system of this form if $y_{\mathcal{F},u}(1) = \xi$. Define the space of solutions of a task T for a given system $\mathcal{F}$ as

$$\begin{aligned} \mathcal{U}_T &= \{u \in \mathcal{U} | u \text{ achieves } T\} \\ &= \{u \in \mathcal{U} | \phi_i(y_{\mathcal{F},u}(t_i)) \\ &= \xi_i, i = 1, \ldots, n\} \subset \mathcal{U} \end{aligned} \tag{4}$$

The aim of motor control is to compute some input function $u(\cdot)$ which achieves the task T in the system $\mathcal{F}$.

1) A priori, it is not clear that a solution exists, i.e. that $\mathcal{U}_T \neq \emptyset$.
2) In general, a solution will not be unique at all. In fact, there are probably infinitely many solutions generically, since we only prescribe the output values at discrete points in time.

If you have an ascending chain of progressively harder tasks

$$T_1 \subset T_2 \subset \cdots \subset T_n, \tag{5}$$

you get a *descending* chain of solution spaces

$$\mathcal{U}_{T_1} \supset \mathcal{U}_{T_2} \supset \cdots \supset \mathcal{U}_{T_n}. \tag{6}$$

The smaller $\mathcal{U}_T$, the harder it becomes to find a solution $u \in \mathcal{U}_T$ among the functions in $\mathcal{U}$.

4. A freezing of parts of the system, as described in [4], corresponds to shrinking the space of inputs $\mathcal{U}$. Naturally, this will simplify the search for a solution.

In contrast to the cost method, another way to simplify the computation of a solution u is a reduction of the dimension S of state space. This is achieved by means of a time-dependent linear projection operator $P(t) : \mathbb{R}^S \to A$, where $A \subset \mathbb{R}^S$ is a linear subspace of dimension k. Such a projection operator gives a rise to a modified system $P\mathcal{F}$:

$$y = \bar{h}(z), \quad \dot{z} = \bar{f}(z) + \bar{g}(u), \tag{7}$$

with $z(t) = P(t)x(t)$. One can then compute an input u that achieves T for this modified system. If P is well-chosen, such u will approximately achieve T for the original system.

Now, a *reduced dimensionality measure* $\mathcal{D}$ is a map

$$\begin{aligned} \mathcal{D}: \quad \Omega &\longrightarrow \quad \mathbb{Z}^+ \\ \mathcal{F} &\longmapsto \quad \mathcal{D}(\mathcal{F}) = \mathcal{D}_I \end{aligned}$$

such that $1 \leq \mathcal{D}_I \leq S$, i.e. an integer dimensionality lower than the full dimensionality of the system. It should quantify the effective dimensionality of the system, in so far as that a projection operator P will have target dimension $k \geq \mathcal{D}_I$.

Given a reduced dimensionality measure $\mathcal{D}$, we will consider a *dimensional change*, which is defined to be a list

$$\begin{aligned} \mathcal{D}_C &= (\mathcal{D}_1, \ldots, \mathcal{D}_m), \\ \text{where } \mathcal{D}_i &= \mathcal{D}(\mathcal{F}_i), \\ \mathcal{F}_i &= (f_i, g_i, h_i, \mathcal{U}_i) \in \Omega \end{aligned} \tag{8}$$

where $\mathcal{D}_C$ is ordered (can be ascending or descending as described in the next section), and the change in dimensionality is due to a small change in either of the three components of the dynamical system, the input $\delta g(\cdot)$, the natural dynamics $\delta f(\cdot)$, or the output $\delta h(\cdot)$, such that $f_{i+1} - f_i = \delta f$, $g_{i+1} - g_i = \delta g$, $h_{i+1} - h_i = \delta h$.

A. Dimensional Change Problem

Consider a chain of tasks as in Eq. 5, and the problem of learning, i.e. finding a solution for, each of the tasks in order, by modying the system $\mathcal{F}$. The problem is, then, to find a dimensional change $\mathcal{D}_C$ in similar form as (5), such that at each stage i, the solution for T_i can tractably be found in the system $\mathcal{F}_i$. In other words, you slightly modify the system at each step to facilitate learning of the more difficult task, finding a chain of solutions $u_i \in (\mathcal{U}_i)_{T_i}$. We look at dimensional changes of a specific form. Most commonly, the only change is in the state dynamics f_i, such that $\delta g = 0$, $\delta h = 0$ at each step. Alternatively, one can hold the f and h fixed and modify the response to the input with changes δg.

Let us consider 2 such subproblems and pose it in the form of Eq. 8.

B. Dimensional Change in Development

During development it is ideal if we begin with some low value of dimensionality and progressively utilise greater and greater dimensions of the musckulo-skeletal system ( ). Let us therefore pose this problem by the set, $\mathcal{D}_C$, from Eq. 8 by $\mathcal{D}_{i+1} \geq \mathcal{D}_i$, i.e. an increasing dimensionality where $\mathcal{D}_1$ is the minimum dimensionality required to carry out a nonzero subset of the tasks $\mathcal{T}$ for some bounded error ϵ.

C. Dimensional Change in Task Learning

During task learning, it is ideal if we begin with the full dimensionality of the system and instead progressively utilise fewer and fewer dimensions of the musckulo-skeletal system ( ). Let us therefore pose this problem by the set, $\mathcal{D}_C$, from Eq. 8 by $\mathcal{D}_{i+1} \leq \mathcal{D}_i$, i.e. an decreasing dimensionality where $\mathcal{D}_m$ is the minimum dimensionality required to carry out a given task $\mathcal{T}_I$ for some bounded error ϵ.

D. Dimensionality Measured using Hankel Singular Values

Let us now examine dimensionality measured using Hankel Singular Values. HSVs can be defined as the vector,

$$\sigma_{HSV}(\mathcal{F}(f(.), g(.), h(.))) = [\sigma_1, \ldots, \sigma_N]^T, \quad (9)$$

where each singular value $\sigma_i \in \mathbb{R}^+$. The Hankel Singular values may be defined in control terms as the product of the controllability and observability gramian of the system $\mathcal{F}$ (explained elsewhere). Since most conventional algorithms for HSV synthesis produce an ordered set (in descending order), we can assume that $\sigma_i \leq \sigma_{i+1}$, where, $1 \leq i \leq N-1$

In order to derive a dimensionality measure of the form of Eq.8, first we redefine each of the individual HSVs by,

$$\tilde{\sigma_i} = \frac{\sum_{j=1}^{i} \sigma_j}{\sum_{i=1}^{N} \sigma_i}, \quad (10)$$

thus the new vector $\tilde{\sigma}_{HSV}$ is the normalised cumulative sum[1] of σ_{HSV}. We now employ a threshold to define the reduced dimensionality by,

$$\mathcal{D}_{\mathcal{HSV}}(\mathcal{F}) = \begin{cases} k & \text{if there exists } \tilde{\sigma}_k \leq t_r, \\ 1 & \text{otherwise} \end{cases} \quad (11)$$

[1]The redefinition to a cummulative sum, allows a monotonically decreasing slope in the graphical visualisation of σ_{HSV}, this is discussed again later.

Parameter	Value
m_1 (link 1 mass)	0.5 kg
m_2 (link 2 mass)	0.5 kg
l_1 (link 1 length)	0.4 m
l_1 (link 1 length)	0.4 m

TABLE I
PARAMETERS FOR THE SIMULATED ARM MODEL

where the threshold t_r is given by $t_r \in \mathbb{R}^+$, $t_r \leq 1$, and k is given by, $k \in \mathbb{Z}^+$, and $1 < k \leq N$. For the rest of this work, unless explicitly stated otherwise, $\mathcal{D}_I$ refers to $\mathcal{D}_{HSV}$.

IV. Experiments

The dimension change problem was studied using a simulated model of a limb. The limb is modelled as a 2 link planar kinematic chain with joint compliance. The actuation is through muscles which respond as first order dynamical systems to activation inputs. The torques applied to the joints result in its motion in space away from the equilibrium.

The parameters of the model are chosen as follows :

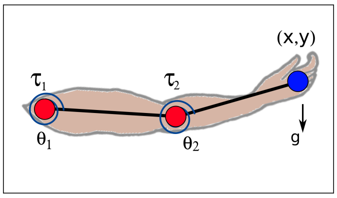

Fig. 1. Model of the limb used for the experiments. The torques are generated by muscles which are first-order in their response to activation. The output is the position at the end of the chain. The joints incorporate passive compliance and damping.

V. Results

The dimensional change problem was

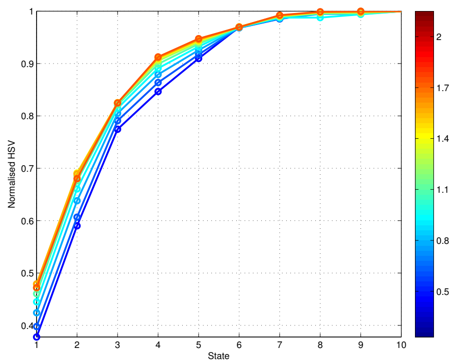

Fig. 2. Change in HSVs with change in joint damping.

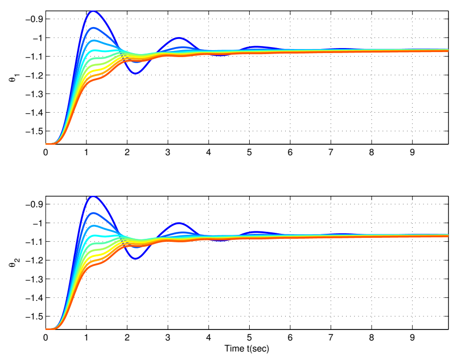

Fig. 3. Trajectory of the joint angles in performing reaching behaviours - variation with joint damping.

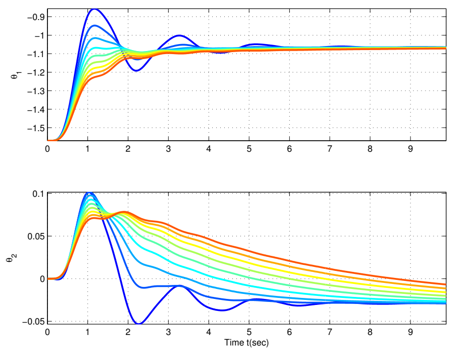

Fig. 4. Endpoint Trajectories with variations in dimensionality.

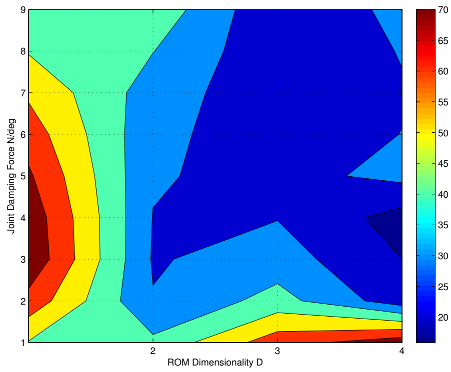

Fig. 5. The reaching error is dependent on joint damping and dimensionality of the reduced order model.

VI. Discussion

Acknowledgement

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Appendix G

Curriculum Vitae

Naveen Kuppuswamy

Curriculum Vitae

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Research Interests : bio-inspired control, motor primitives/synergies, optimal motor control, model/control order reduction

Education

2009-2013 **PhD in Artificial Intelligence**, *(magna cum laude) University of Zürich*, Switzerland.
Thesis Title : *Exploiting reduced dimensionality in the design and control of embodied systems*
Supervisor : Prof. Dr. Rolf Pfeifer
Marie Curie Fellow

2005–2007 **MS in Electrical Engineering and Computer Science**, *Korea Advanced Institute of Science and Technology (KAIST)*, Daejeon, South Korea.
Thesis Title : *Nonlinear inverse dynamic control of an omnidirectional mobile robot using slip rolling modes*
Supervisor : Prof. Dr. Jong-Hwan Kim
Korean Research Foundation (KRF) Scholar

2001-2005 **BE in Instrumentation and Control Engineering**, *Anna University*, Chennai, India.

Experience

Academic Research

2009–2013 **EU Projects OCTOPUS(IP), ROBOTDoC(ITN), AMARSi(IP)**, *AI Lab*, UZH, Switzerland.
Worked on reduced dimension control, design principles for modular control architectures, and the learning of motor primitives (AMARSi and ROBOTDoC), as well as on continuum robot control and low-level control hardware development (OCTOPUS). A part of the work was carried out at the Centre for Robotics and Neural Systems, Plymouth University, UK.

2005–2007 **Information Technology Research Center (ITRC) Funded Projects**, *Robot Intelligence Technology Lab*, KAIST, Daejeon, South Korea.
Worked on the development and control of an omnidirectional football robot; on movement planning and control of bipedal robots; and on ubiquitous robots and cognitive control architectures with episodic memory.

Industry

2007-2008 **Engineer R&D**, *Yujin Robot Co. Ltd.*, Seoul, South Korea.
Worked on service robot middleware and computer vision for human robot interaction.

Academic Experience

Publications

- Coauthored 4 journal papers (1 under review and 2 to be submitted shortly), 1 book chapter, 15 conference papers, and 3 posters. Full published list available at Google Scholar

.

Teaching Assistance

- Introduction to AI(ShanghAI lectures), Fall 2012, Fall 2009
- Formal Methods in Computer Science II, Spring 2007
- Bio-Inspired Robotics, Spring 2007

Student Supervision

- Supervised 4 bachelor's and 2 master's theses, and 3 semester projects

Summer Schools

- EMBODYi Projects Summer School on Embodied Intelligence, Livorno, Italy (September 2010).
- Veni Vidi Vici (iCub summer school), Sestri Levante, Italy (August 2011).
- Summer School on Impedance Control, Fraueninsel Chiemsee, Germany (July 2011).
- Multiple events as part of the RobotDoc network training activities (September 2009 - August 2013)

Technical Skills

Languages	C, C++, Python,MATLAB Script, LaTeX, HTML
Embedded	Atmel AVR Series, Arduino, Raspberry Pi
OS	Linux (various flavours), Windows XP
Software	Solidworks, Inkscape, Gimp, GNU Octave, Matlab, Eclipse, KiCad

Achievements and Activities

- Served as Chairman at the *Science Cafè* at the *Robots on Tour* event, Cafè Sphères, Zürich, on 8^{th} March, 2013.
- Served as Publicity Chair at the *Postgraduate Conference on Robotics and Development of Cognition (ROBOTDoC-PhD)*, a satellite event of the International Conference on Artificial Neural Networks (ICANN 2012), at the University of Lausanne, Switzerland, on $10 - 12^{th}$ September 2012.
- Served as the Coordinator of the Brown bag lecture series conducted at the AI Lab and at ETH Zurich from 2009 until present.
- Recepient of the *Best Documentation Award* at the 13^{th} *International Microrobot Maze Contest*, held in Nagoya, Japan in October 2004.

Personal

Birthplace	Chennai, India.
Birth date	19^{th} October, 1983.
Languages	Tamil (Fluent), English (Fluent), Hindi (Expert), Korean (Intermediate), German (Intermediate).
Hobbies	Comic and Graphic Art, Cooking, Music (Guitar/Vocals), Hobby Electronics.

Selected Publications

Journals and Book Chapters

[J1] J. Carbajal and N. Kuppuswamy. Magneto-mechanical actuation model for fin-based locomotion. *International Journal of Design & Nature and Ecodynamics*, 8:1–10, 2013.

[J2] J.-H. Kim, C.-H. Lee, K.-H. Lee, and N. S. Kuppuswamy. Evolutionary generation of artificial creature's personality for ubiquitous services. In *Advances in Metaheuristics for Hard Optimization*, pages 263–292. Springer, 2008.

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Posters

[P13] N. Kuppuswamy and C. Alessandro. Impact of body parameters on dynamic movement primitives for robot control. *Procedia Computer Science*, 7:166–168, 2011.

[P14] N. Kuppuswamy and J.-P. Carbajal. Learning a curvature dynamic model of an octopus-inspired soft robot arm using flexure sensors. *Procedia Computer Science*, 7:294–296, 2011.